UNIT 1: KINEMATICS

Applying Kinematic Equations

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$\Delta x = \frac{1}{2}(v_0 + v)t$$

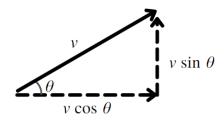
- These kinematic equations are only applicable of the acceleration is constant. <u>If acceleration is not constant, you cannot use these equations.</u>
- Near the surface of the Earth, acceleration is always downward at $g = 9.8 \text{ m/s}^2$
- Objects at maximum height have a vertical velocity of 0.
- Objects that are at <u>rest</u>, <u>dropped</u> or <u>released</u> have a velocity of 0.
- Objects at <u>constant speed</u> have an acceleration of 0.

Graphs of Motion

- Slope of position vs time = velocity
- Slope of velocity vs time = acceleration
- Area under velocity vs time = displacement
- Area under acceleration vs time = change in velocity

Projectile Motion

• Decomposing vectors:

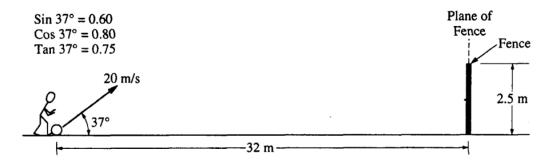


- t is the only common variable between horizontal and vertical components
- At maximum height, the vertical component of velocity is 0
- Make sure you are consistent with the sign (+/-) and direction of each variable.

Problem Solving Steps

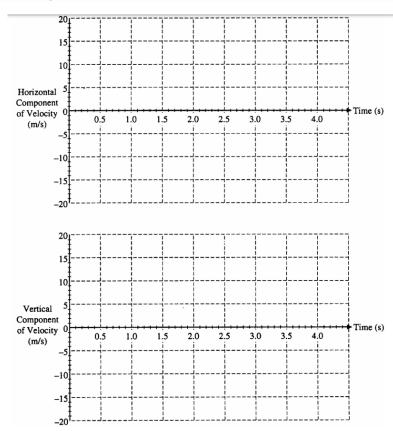
- 1) List all 5 kinematic variables for both horizontal and vertical directions.
- 2) Assign positive and negative directions for horizontal and vertical directions.
- 3) Horizontal Direction: $v_0 = v$ and a = 0. The only relevant equation is $\Delta x = v_0 t$
- 4) Vertical Direction: acceleration is downward at $g = 9.8 \frac{m}{c^2}$.
- 5) Decompose velocities into horizontal and vertical components.
- 6) Identify the unknowns and what you're trying to solve for.
- 7) Apply kinematic equations for vertical and horizontal motion.

Example FRQ



Note: Diagram not drawn to scale.

- 1994B1 (modified) A ball of mass 0.5 kilogram, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown above. The velocity of the ball as it leaves the kicker's foot is 20 meters per second at an angle of 37° above the horizontal. The top of the fence is 2.5 meters high. The ball hits nothing while in flight and air resistance is negligible.
- a. Determine the time it takes for the ball to reach the plane of the fence.
- b. Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?
- c. On the axes below, sketch the horizontal and vertical components of the velocity of the ball as functions of time until the ball reaches the plane of the fence.



UNIT 2: FORCES

Newton's Laws of Motion

 $F_{\text{net}} = ma$

- Newton's 1st Law: An object can only change its velocity when a force acts on it
- Newton's 2^{nd} Law: $F_{net} = ma$
- Newton's 3rd Law: If an object causes a force, and equal and opposite force applies to the object itself.

Forces Rules for Free Body Diagrams

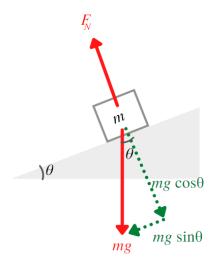
- Gravity always points down toward the center of the Earth
- All other forces require physical contact. Examine what else is touching the object.
- Normal force is from a surface and pushes outward from the surface
- Tension force is from a rope and always pulls
- Frictional forces are parallel to a surface and oppose motion
 - Kinetic friction
 - $F_f = \mu_k F_N$
 - Occurs when two surfaces that are sliding against each other.
 - The direction of the force is against the motion.
 - Static friction
 - $F_f \leq \mu_S F_N$
 - Static friction is a reactive force, and the object will only slide when the maximum static friction is overcome ($F_{fmax} = \mu_S F_N$)
 - Direction is difficult to determine because there's no sliding. The easiest way is to see which direction the object would move if there were no friction. Static friction would oppose that direction.
- Spring forces will pull or push back to the relaxed length (natural length)
 - \circ $F_{sp} = kx$
 - Hooke's Law describes an ideal spring where the force is proportional to how much you stretch or compress it.
 - o The spring always wants to return to its relaxed length. So a compressed a spring will provide a force outward, and a stretched spring will pull inward.
 - \circ The spring constant k is a physical property of the spring (units of N/m). The larger the spring constant, the "stiffer" the spring.

How to Solve Free Body Diagrams

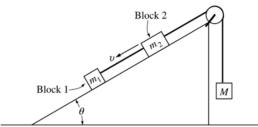
- 1) Draw all forces acting on the object (or system)
- 2) Decompose force vectors into the horizontal and vertical components
- 3) Assign positive and negative directions
- 4) Apply F = ma in both the horizontal and vertical directions

Inclined Planes

- The acceleration on an inclined plane is typically either up the ramp, down the ramp, or zero. Thus, we set the "x-axis" so that it's parallel to the incline, and the "y-axis" as perpendicular to the incline.
- We decompose all vectors onto this rotated x and y axis

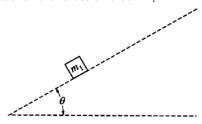


Example FRQ



2000B2. Blocks 1 and 2 of masses m_1 and m_2 , respectively, are connected by a light string, as shown above. These blocks are further connected to a block of mass M by another light string that passes over a pulley of negligible mass and friction. Blocks 1 and 2 move with a constant velocity v down the inclined plane, which makes an angle θ with the horizontal. The kinetic frictional force on block 1 is f and that on block 2 is 2f.

a. On the figure below, draw and label all the forces on block m₁.



Express your answers to each of the following in terms of m₁, m₂, g, θ, and f.

- b. Determine the coefficient of kinetic friction between the inclined plane and block 1.
- c. Determine the value of the suspended mass *M* that allows blocks 1 and 2 to move with constant velocity down the plane.
- d. The string between blocks 1 and 2 is now cut. Determine the acceleration of block 1 while it is on the inclined plane.

UNIT 3: CIRCULAR MOTION & GRAVITY

Centripetal Acceleration

$$a_c = \frac{v^2}{R}$$

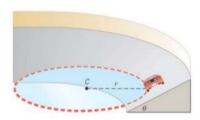
- Centripetal acceleration is when the acceleration causes the velocity to change directions
- Centripetal acceleration is always perpendicular to the velocity and points towards the center of the circle
- Uniform Circular Motion refers to objects moving in a circle at constant speed.
- The period of a uniform circular motion (T) can be found by $T = \frac{2\pi R}{v}$
- The centripetal force is the net force causing the centripetal acceleration but it is <u>not</u> a new force to add to a free body diagram.

Solving Free Body Diagrams

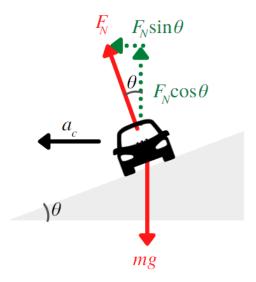
- Draw the free body diagram using the exact same rules as usual. There are no new forces to draw.
- When applying F = ma, you replace a with $\frac{v^2}{R}$ if it's centripetal acceleration in that direction.
- Because $\frac{v^2}{R}$ is always positive, make sure you attach a negative sign if the direction towards the center of the circle is the negative direction.

Banked Turns

• When vehicles turn, static friction usually provides the centripetal acceleration. However, sometimes that's not enough so that the car is inclined and the normal force can provide the centripetal acceleration.



- An ideal banking is where the angle of the incline matches the speed the vehicle is moving so that no friction is necessary to keep the vehicle turning.
- When drawing the free body diagram, we keep the x-axis horizontal because the centripetal acceleration is in the horizontal plane.



• When static friction is present, it can be difficult to determine the direction. It should be directed down the ramp when a car is going faster and needs more centripetal acceleration. It should be directed up the ramp when a car is going slower and needs less centripetal acceleration.

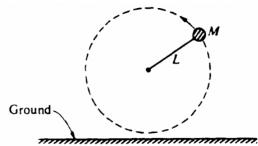
Newton's Law of Gravitation

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$g = G \frac{m}{r^2}$$

- When objects are far from the surface of the planet, the force of gravity is more accurately described by Newton's Law of Gravitation.
- The gravitational field is the acceleration due to gravity. We have typically used 9.8 m/s² for near the surface of the Earth, but it can be a smaller quantity as you get further from the planet, or different on other planets with different masses and sizes.
- The direction of the force of gravity is still the same: towards the center of the Earth (or whatever object is causing the gravity).
- The density of an object is the ratio of the mass to the volume $(\rho = \frac{m}{V})$

Example FRQ



1984B1. A ball of mass M attached to a string of length L moves in a circle in a vertical plane as shown above. At the top of the circular path, the tension in the string is twice the weight of the ball. At the bottom, the ball just clears the ground. Air resistance is negligible. Express all answers in terms of M, L, and g.

- a. Determine the magnitude and direction of the net force on the ball when it is at the top.
- b. Determine the speed v_0 of the ball at the top.

The string is then cut when the ball is at the top.

- c. Determine the time it takes the ball to reach the ground.
- d. Determine the horizontal distance the ball travels before hitting the ground.

UNIT 4: WORK & ENERGY

Work

$$W = F_{||}d = Fd\cos\theta$$

- Work is a scalar quantity, so it has no direction. However, it can be positive or negative (but the sign does <u>not</u> indicate direction)
- To compute work, you take the component of force that is parallel to the displacement and multiply them together.
- If the force component is opposite the direction of displacement, the work is negative. If the force component is in the same direction as displacement, the work is positive.
- If the force is perpendicular to the displacement, it does no work.
- In a graph of Force vs Displacement, the area under the curve is the work.

Work & Energy (Conservation of Energy)

$$W = \Delta E = E_f - E_0$$

- Like work, energy is a scalar quantity. It can be positive or negative, but it does not have direction.
- Work by an external force causes a change in the energy of a system.
- If there is no external work on the system, then we can apply conservation of energy.

Kinetic Energy

$$KE = \frac{1}{2}mv^2$$

Kinetic energy is the energy of motion. If an object has speed, it has kinetic energy.

Gravitational Potential Energy (Near Surface of Earth)

$$U_{grav} = mgh$$

- We only include gravitational potential energy if the Earth is part of the system. Thus, gravity is an internal force and doesn't do <u>external</u> work on the system.
- Near the surface of the Earth, the gravitational potential energy is proportional to the height (mgh).
- Potential energy is relative to a specific height. So we usually set the lowest point in the problem to a height of 0 in order to calculate the potential energy.
- We calculate the total energy by summing the kinetic energy of every object and any gravitational potential energy (assuming the Earth is part of the system)

Gravitational Potential Energy (Far from Surface of Earth)

$$U_{grav} = -\frac{Gm_1m_2}{r}$$

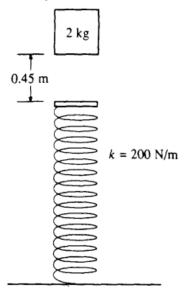
- When we get further from the surface of a planet, the gravitational force is no longer equal to mg.
- We use this gravitational potential equation between two masses that are separated by great distances
- Every pair of masses has a potential energy. So the potential energy of a system is the sum of every pair of masses.

Spring Potential Energy

$$U_{spring} = \frac{1}{2}kx^2$$

- To stretch or compress a spring takes work (must apply a force over displacement).
- That work is stored as potential energy in the spring.
- If a spring is compressed or stretched, then there's energy stored in the spring.

Example FRQ



C1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

- a. Determine the speed of the block at the instant it hits the end of the spring.
- b. Determine the force in the spring when the block reaches the equilibrium position
- c. Determine the distance that the spring is compressed at the equilibrium position
- d. Determine the speed of the block at the equilibrium position
- e. Is the speed of the block a maximum at the equilibrium position, explain.

UNIT 5: LINEAR MOMENTUM

Impulse-Momentum

$$p = mv$$

$$F\Delta t = \Delta p = m\Delta v$$

- Momentum (denoted with the letter p) is defined to be the product of mass and velocity.
- Momentum is a vector quantity (has magnitude and direction).
- Impulse is defined to the be product of Force and time.
- Rearranging Newton's Second Law gives us the Impulse-Momentum Theorem
- An impulse is required to cause a change in the momentum of an object (or system)
- The area under the curve for a force vs time graph is equal to the impulse

Center of Mass

$$x_{\rm cm} = \frac{\sum m_{\rm i} x_{\rm i}}{\sum m_{\rm i}}$$

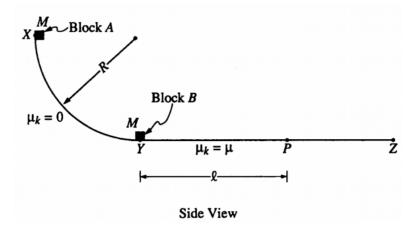
$$v_{\mathrm{cm}} = \frac{\sum m_{\mathrm{i}} v_{\mathrm{i}}}{\sum m_{\mathrm{i}}}$$

- When we combine multiple objects in a system, the forces interacting are internal to the system.
- The center of mass can be calculated by taking a weighted sum of the position of the objects and their mass.
- The velocity of the center of mass can be calculated in a similar way.

Conservation of Momentum

- If there's no external impulse, then there's no change in momentum (conservation of momentum).
- In a collision, if both objects are part of the system, then there's no external impulse (forces act on each other but would be internal), so there's no change in total momentum.
- To apply conservation of momentum, you calculate the total momentum before the collision and set it equal to the total momentum after the collision.
- Note that the center of mass of the system will maintain the same velocity before and after the collision because the forces are internal to the system.
- Elastic vs Inelastic Collisions
 - Elastic collisions are collisions where the total kinetic energy of the system stays constant before and after the collision.
 - o Inelastic collisions are collisions where the total kinetic energy of the system changes before and after the collision.
- Conservation of momentum applies in two-dimensions because it is a vector quantity
- Momentum may be conserved in one direction, but not in another depending on whether there's an external impulse in that direction.

Example FRQ



1994B2. A track consists of a frictionless arc XY, which is a quarter-circle of radius R, and a rough horizontal section YZ. Block A of mass M is released from rest at point X, slides down the curved section of the track, and collides instantaneously and inelastically with identical block B at point Y. The two blocks move together to the right, sliding past point P, which is a distance L from point Y. The coefficient of kinetic friction between the blocks and the horizontal part of the track is μ Express your answers in terms of M, L, μ , R, and g.

- a. Determine the speed of block A just before it hits block B.
- b. Determine the speed of the combined blocks immediately after the collision.
- c. Assuming that no energy is transferred to the track or to the air surrounding the blocks. Determine the amount of internal energy transferred in the collision
- d. Determine the additional thermal energy that is generated as the blocks move from Y to P

UNIT 6: SIMPLE HARMONIC MOTION

Definition

- Simple harmonic motion is an oscillating motion that has a restoring force (i.e. the net force points towards equilibrium) that is proportional to the displacement from equilibrium.
- Characteristics of simple harmonic motion
 - o The motion is periodic
 - o The position, velocity, and acceleration are sinusoidal curves
 - o The total energy (kinetic and potential) remains constant
- There are 2 basic scenarios we cover in AP Physics 1: pendulum and mass on a spring

Pendulum

$$T = 2\pi \sqrt{\frac{l}{g}}$$

- Pendulums are only approximately simple harmonic motion for small angles (less than 15°)
- The pendulum period is dependent only on the length of the string and the acceleration due to gravity. It is not affected by the mass.
- At the maximum height, there's no kinetic energy and all of the energy is gravitational potential energy.
- Equilibrium is at the minimum height, and it has the maximum kinetic energy at that point (thus, is moving fastest).

Horizontal Mass-Spring System

$$T = 2\pi \sqrt{\frac{m}{k}}$$

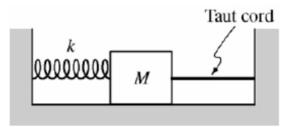
- The period of the mass-spring system is only dependent on the mass of the block and the spring constant. It does not depend on the amplitude (i.e. the maximum displacement).
- The amplitude of motion is the maximum compression/stretch of the spring.
- At maximum compression and stretch, the block has no kinetic energy (v = 0) and all of the system's energy is stored as spring potential energy.
- The point where the spring is relaxed is the equilibrium position ($F_{net} = 0$), and the block has the maximum kinetic energy and speed.

Vertical Mass-Spring System

$$T = 2\pi \sqrt{\frac{m}{k}}$$

- The period of the mass-spring system is the same as for horizontal springs.
- The amplitude of motion is the maximum compression/stretch of the spring.
- At maximum stretch, the system has the lowest gravitation potential energy, no kinetic energy (speed is 0), and the maximum spring potential energy.
- At maximum compression, the system has the highest gravitation potential energy, no kinetic energy, and some spring potential energy.
- The point where the net force is zero is the equilibrium point. This is <u>not</u> when the spring is relaxed, but the spring force is balanced by the gravitational force. The kinetic energy is maximum here.

Example FRQ



One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.

(a) On the dot below that represents the block, draw and label a free-body diagram for the block.

•

(b) Calculate the distance that the spring has been stretched from its equilibrium position.

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.

- (c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position.
- (d) Calculate the time after the cord breaks until the block first reaches its position furthest to the left.
- (e) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.

UNIT 7: ROTATIONAL MOTION

Rotational Kinematics

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{\omega_0 + \omega}{2}t$$

$$a = \alpha R$$
, $v = \omega R$, $d = R\Delta \theta$

- Rotational kinematic equations are identical to kinematic equations but with rotational quantities
- Rotational quantities are still vectors, but we describe the direction as "clockwise" vs "counterclockwise". One direction is positive, and the other is negative.
- Linear motion variables can be related to the angular/rotational variables.

Torque

$$\tau = r_{\perp}F = rF_{\perp} = rF\sin\theta$$

$$\tau_{net} = I\alpha$$

- Free body diagrams are identical, except now we care about where the force is applied.
 - o Gravity acts at the center of mass
 - o All other forces act where they are making physical contact
- Torque calculation requires us to identify an axis of rotation.
- Torque is a vector quantity so it also follows the clockwise/counterclockwise direction.
- Torque causes angular acceleration.

Solving Problems with Torque

- 1) Draw the free body diagram and place the force where it acts.
- 2) Calculate the torque of each force by drawing the r-vector from axis of rotation to where the force is applied. Then sum up the torques to calculate τ_{net}
- 3) Apply $\tau_{net} = I\alpha$

Rotational Inertia

 $I = mr^2$ (for a point mass)

- The more rotational inertia an object has, the harder it is to rotate.
- Rotational inertia depends on both the mass and how the mass is distributed.
- The total rotational inertia of a system is the sum of the rotational inertia of every object or mass in the system.

Rotational Kinetic Energy

$$KE_{rot} = \frac{1}{2}I\omega^2$$

- Rotational Kinetic Energy is still an energy of motion but caused by an object spinning around an axis.
- Translational Kinetic Energy is the energy of motion due to the center of mass moving at a speed.
- An object can have <u>both</u> rotational kinetic energy and translational kinetic energy (e.g. a ball is rolling down a hill is both moving and spinning).
- All normal energy principles apply (work causes change in energy, and conservation of energy).

Rolling

- Rolling without slipping means...
 - There's no kinetic friction because the object is not rubbing against the surface.
 - We can apply the relationship between angular and linear motion $v = r\omega$ and $a = r\alpha$
 - There must be some static friction if the object has angular acceleration (e.g. speeding up or slowing down)
- Static friction usually acts as the torque to cause something to roll faster or slower. It doesn't do work on the object, so there's no change in energy.

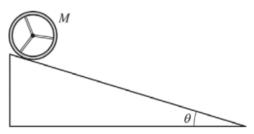
Conservation of Angular Momentum

 $L = I\omega$ (for a system)

 $\tau \Delta t = \Delta L$

- Changes in angular momentum must be caused by an external angular impulse (i.e. external torque over time)
- If there's no external torque, then angular momentum is "conserved" (i.e. it is kept constant)
- Conservation of angular momentum is very powerful and is used in collisions (just like linear momentum), however it's also applicable in any cases where two objects are interacting to cause a change in rotation.
- Conservation of angular momentum is also useful in scenarios with single objects, but the rotational inertia is changing (e.g. a diver who spins before landing in the water, or a figure skater who spins faster by bringing their arms in)

Example FRQ

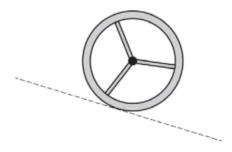


1. (7 points, suggested time 13 minutes)

A wooden wheel of mass M, consisting of a rim with spokes, rolls down a ramp that makes an angle θ with the horizontal, as shown above. The ramp exerts a force of static friction on the wheel so that the wheel rolls without slipping.

(a)

i. On the diagram below, draw and label the forces (not components) that act on the wheel as it rolls down the ramp, which is indicated by the dashed line. To clearly indicate at which point on the wheel each force is exerted, draw each force as a distinct arrow starting on, and pointing away from, the point at which the force is exerted. The lengths of the arrows need not indicate the relative magnitudes of the forces.



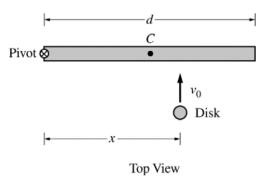
ii. As the wheel rolls down the ramp, which force causes a change in the angular velocity of the wheel with respect to its center of mass?

Briefly explain your reasoning.

- (b) For this ramp angle, the force of friction exerted on the wheel is less than the maximum possible static friction force. Instead, the magnitude of the force of static friction exerted on the wheel is 40 percent of the magnitude of the force or force component directed opposite to the force of friction. Derive an expression for the linear acceleration of the wheel's center of mass in terms of M, θ, and physical constants, as appropriate.
- (c) In a second experiment on the same ramp, a block of ice, also with mass *M*, is released from rest at the same instant the wheel is released from rest, and from the same height. The block slides down the ramp with negligible friction.

i.	Which object, if either, reaches the bottom of the ramp with the greatest speed?					
	Wheel	Block	Neither; both reach the bottom with the same speed.			
	Briefly explain your answer, reasoning in terms of forces.					

ii. Briefly explain your answer again, now reasoning in terms of energy.



3. (12 points, suggested time 25 minutes)

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass $m_{\rm disk}$ toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a)	1.1		an the disk. To give the rod as much angular speed as possible d to the left of point C , at point C , or to the right of point C ?	
	To the left of <i>C</i>	At <i>C</i>	To the right of <i>C</i>	

Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{\rm disk} \, x \, v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?

____ Yes ____ No

Briefly explain your reasoning without deriving an equation for ω .

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{Ixv_0}{m_{\rm disk}d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}} x^2$						
	angular momentum with respect to the pivot is $m_{\text{disk}} v_0 x$. Derive an equation for the postcollision angular					
	speed ω of the rod. Express your answer in terms of d , $m_{\rm disk}$, I , x , v_0 , and physical constants, as appropriate.					
(e)	(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounce backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?					
	Greater than Less than Equal to					
	Briefly explain your reasoning.					
(e)	backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it? Greater than Less than Equal to					