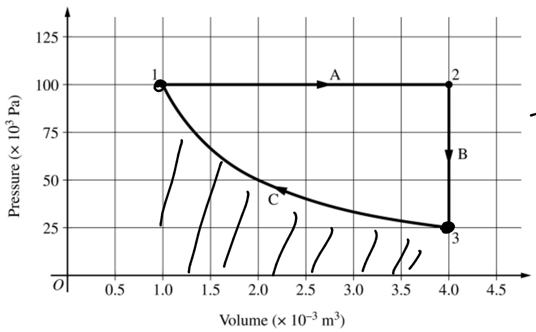
2021

Sunday, May 09, 2021 12:09 PM



$PV = nRT$

$\frac{P}{T} = \text{const}$

$PV = nRT$

$W = P\Delta V$

$= -100 \times 10^{3}\,\text{Pa} \times (3 \times 10^{-3})$

$\boxed{W = -300\text{J}}$

1. (10 points, suggested time 20 minutes)

A sample of ideal gas is taken through the thermodynamic cycle shown above. Process C is isothermal.

(a) Consider the portion of the cycle that takes the gas from state 1 to state 3 by processes A and B. Calculate the magnitude of the following and indicate the sign of any nonzero quantities.

- The net change in internal energy ΔU of the gas
- The net work W done on the gas
- The net energy Q transferred to the gas by heating

$\Delta U =$

$U = \frac{3}{2} nRT$

$U = \frac{3}{2} PV$

$\Delta U = W + Q$

$\Delta U = \frac{3}{2}(4 \times 10^{-3})(25 \times 10^{3}\,\text{Pa}) - \frac{3}{2}(100 \times 10^{3}\,\text{Pa})(1 \times 10^{-3}\,\text{m}^3)$

$\Delta U = 0$

$Q = \Delta U - W$

$= 0 - -300\text{J}$

$Q = 300\text{J}$

(b) Consider isothermal process C.

i. Compare the magnitude and sign of the work W done on the gas in process C to the magnitude and sign of the work in the portion of the cycle in part (a). Support your answer using features of the graph.

W_c has a positive sign because the volume of gas is decreasing (going left on the curve).

area under curve for C < area under curve for A

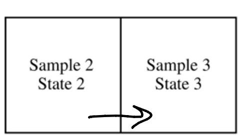
$\Rightarrow |W_c| < |W_{AB}|$

ii. Explain how the microscopic behavior of the gas particles and changes in the size of the container affect interactions on the microscopic level and produce the observed pressure difference between the beginning and end of process C.

isothermal ⟹ constant temperature ⟹ constant ~~same~~ avg. kinetic energy

volume decreases which means collisions with the sides of the container occur more frequently because the walls are closer together.

this increased collisions results in higher pressure.

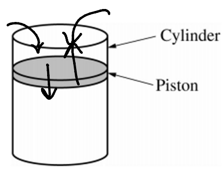


(c) Consider two samples of the gas, each with the same number of gas particles. Sample 2 is in state 2 shown in the graph, and sample 3 is in state 3 shown in the graph. The samples are put into thermal contact, as shown above. Indicate the direction, if any, of energy transfer between the samples. Support your answer using macroscopic thermodynamic principles.

State 2 has a higher temperature than state 3 because volume is the same, but ($PV = nRT$)

State 2 has a higher temperature than State 3 because volume is the same, but state 2 has higher pressure ($PV=nRT$)

By the 2nd law of thermodynamics, heat flows from higher temp. to lower temp. So heat flows from 2 to 3 (to the right)



2. (12 points, suggested time 25 minutes) $\rho = \frac{M}{V}$

A group of students design an experiment to investigate the relationship between the density and pressure of a sample of gas at a constant temperature. The gas may or may not be ideal. They will create a graph of density as a function of pressure. They have the following materials and equipment.

$PV=nRT$

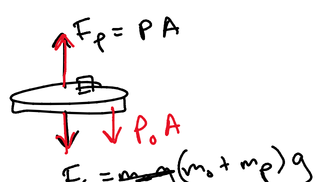
- A sample of the gas of known mass M_g in a sealed, clear, cylindrical container, as shown above, with a movable piston of known mass m_p
- A collection of objects each of known mass m_o
- A meterstick

(a)

i. Describe the measurements the students should take and a procedure they could use to collect the data needed to create the graph. Specifically indicate how the students could keep the temperature constant. Include enough detail that another student could follow the procedure and obtain similar data.

1) place some mass of m_o on the piston
2) let the piston fall and wait for a few minutes for temp of gas to reach room temperature.
3) measure the radius height of the piston w/ the meterstick (record) h
4) repeat ~~width~~ ~~starting~~ at step 1 with an additional mo. repeat several masses.

ii. Determine an expression for the absolute pressure of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.



h = measured height
r = radius of cylinder

P_0 = atmospheric pressure

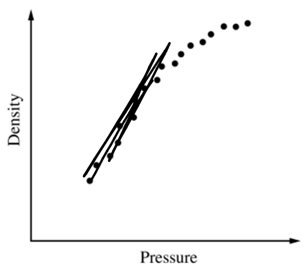
$$P_0 A + (m_o + m_p)g = PA = P\pi r^2$$

$$\boxed{P = P_0 + \frac{(m_o+m_p)g}{\pi r^2}}$$

iii. Determine an expression for the density of the gas in terms of measured quantities, given quantities, and physical constants, as appropriate. Define any symbols used that are not already defined.

$$\text{density} = \frac{M}{V} = \boxed{\frac{M_g}{\pi r^2 h}}$$

$PV = nRT$ $\quad d = \frac{M}{}$



$PV = nRT$

$d = \frac{m}{V}$

$V = \cancel{md}\ \frac{m}{d}$

$P\left(\frac{m}{d}\right) = nRT$

iv. The graph above represents the students' data. Does the data indicate that the gas is ideal? Describe the application of physics principles in an analysis of the graph that can be used to arrive at your answer.

$x \rightarrow \frac{Pm}{nRT} = d \leftarrow y$

ideal gas, density vs pressure should have a linear relationship.

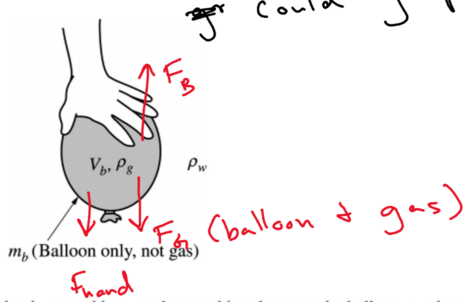
This graph is not linear so at higher pressures, this gas is not ideal

Another group of students propose that the relationship between density and pressure could also be obtained by filling a balloon with the gas and submerging it to increasing depths in a deep pool of water.

(b) Why could submerging the balloon to increasing depths be useful for determining the relationship between the density and pressure of the gas?

increasing depth $\Rightarrow$ increase in pressure and it will reduce the volume.

could graph the changing P & V



(c) The balloon is kept underwater in the deep pool by a student pushing down on the balloon, as shown above. Let V_b represent the volume of the inflated balloon, m_b represent the mass of just the balloon (not including the mass of the gas), ρ_g represent the density of the gas in the balloon, and ρ_w represent the density of the water. Derive an expression for the force the student must exert to hold the balloon at rest under the water, in terms of the quantities given in this part and physical constants, as appropriate.

$$F_G = m_b g + \rho_g V_b g$$

$$F_B = \rho_w V_b g$$

$$F_{hand} + F_G = F_B$$

$$F_{hand} = F_B - F_G = \boxed{\rho_w V_b g - m_b g - \rho_g V_b g}$$

$$g(V_b(\rho_w - \rho_g) - m_b)$$

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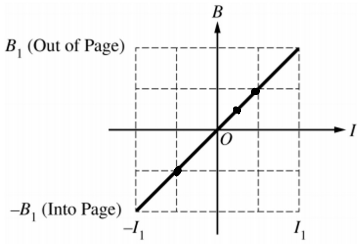
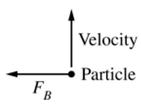


Graph 1

3. (12 points, suggested time 25 minutes)

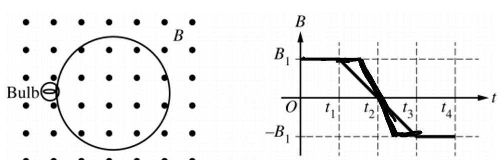
An electromagnet produces a magnetic field that is uniform in a certain region and zero outside that region. The graph above represents the field as a function of the current in the electromagnet, with positive field directed out of the page and negative field directed into the page.



(a) The current in the electromagnet is set at $0.5I_1$. When a charged particle in the region moves toward the top of the page, the force exerted on it by the field is F_B toward the left, as shown above. What changes to the current in the electromagnet could make the magnitude of the force exerted on the particle equal to $2F_B$ and the direction of the force to the right? Support your answer using physics principles.

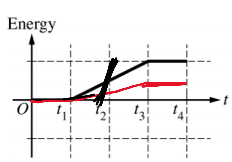
$$F_B = q\bar{v} \times \bar{B}$$

to have F_B flip & double, we need to double $\bar{B}$ and ~~change its~~ reverse it's direction. Because the graph of B vs I is linear, we need to have $I = -I_1$



Graph 2

A circuit is made by connecting an ohmic lightbulb of resistance R and a circular loop of area A made of a wire with negligible resistance. The circuit is placed with the plane of the loop perpendicular to the field of the electromagnet, as shown above on the left. The magnetic field changes as a function of time, as shown in Graph 2. The bulb dissipates energy during the interval $t_1 < t < t_3$. Graph 3 below shows the cumulative energy dissipated by the bulb (the total energy dissipated since $t = 0$) as a function of time.



Graph 3

(b) The original bulb is replaced by a new ohmic lightbulb with a greater resistance, but everything else stays the same. How would the cumulative energy graph for the new bulb be different, if at all, from Graph 3 above? Support your answer using physics principles.

$P = VI$ $\quad |V| = \left|\frac{\Delta\phi}{\Delta t}\right|$ ← staying the same

$P = \frac{V^2}{R}$ $\quad I = \frac{V}{R}$ ↓ $\frac{1}{2}$

R ↑ 2x than

P drops by 2x ⇒ Energy drops by 2x

same shape but y-values are all cut in half.

P drops by 2x ... energy .

same shape but y-values are all cut in half.

(c) The new lightbulb is removed and replaced by the original lightbulb. The magnetic field now changes from $2B_1$ to $-2B_1$ during the same interval $t_1 < t < t_3$. A new cumulative energy graph is created for this situation. How would the new graph be different, if at all, from Graph 3 ? Support your answer using physics principles.

twice the change in B ($\phi = BA$) results in double the voltage. $P = \frac{V^2}{R} \Rightarrow$ power is now 4x as much

energy $\Rightarrow$ 4x

graph still same shape but y-values are 4x

(d) A student derives the following expression for the cumulative energy dissipated by the original bulb during the interval $t_1 < t < t_3$ and with the original change in magnetic field shown in Graph 2.

$$\text{Energy} = \frac{A^2 B_1 R}{4(t_3 - t_1)} \checkmark$$

Whether or not the equation is correct, does the functional dependence of cumulative energy on the elapsed time $(t_3 - t_1)$ make physical sense? Support your answer using physics principles.

$$V = \frac{\Delta\Phi}{\Delta t} = \frac{\Delta B \cdot A}{\Delta t} = \frac{\Delta B \cdot A}{t_3 - t_1}$$

$$P = \frac{V^2}{R} \propto \frac{1}{(t_3 - t_1)^2}$$

$$E = P \cdot t \propto \frac{1}{(t_3 - t_1)^2} \cdot (t_3 - t_1) = \frac{1}{t_3 - t_1}$$

this is a correct relationship

4. (10 points, suggested time 20 minutes)

Light and matter can be modeled as waves or as particles. Some phenomena can be explained using the wave model, and others can be explained using the particle model.

(a) Calculate the speed, in m/s, of an electron that has a wavelength of 5.0 nm.

$$\lambda = \frac{h}{p} = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$$

$$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31}\,\text{kg} \times 5 \times 10^{-9}\,\text{m}}$$

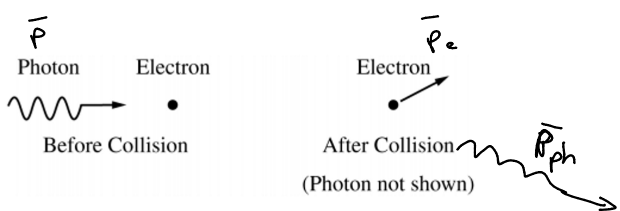
$= \boxed{1.46 \times 10^5 \text{ m/s}}$

(b) The electron is moving with the speed calculated in part (a) when it collides with a positron that is at rest. A positron is a particle identical to an electron except that its charge is positive. The two particles annihilate each other, producing photons. Calculate the total energy of the photons.

$$E_{tot} = m_e c^2 + m_p c^2 + \underbrace{KE_e}_{\frac{1}{2}mv^2} + KE_p$$

$$= \left(9.11\times10^{-31} \times \left(3\times10^8 \tfrac{m}{s}\right)^2\right) \times 2 + 2 \times \left(\tfrac{1}{2} \times 9.11\times10^{-31}\,kg\right)\left(1.46\times10^5 \tfrac{m}{s}\right)^2$$

$$= 1.64\times10^{-13}\ J$$



(c) A photon approaches an electron at rest, as shown above on the left, and collides elastically with the electron. After the collision, the electron moves toward the top of the page and to the right, as shown above on the right, at a known speed and angle. In a coherent, paragraph-length response, indicate a possible direction for the photon that exists after the collision and its frequency compared to that of the original photon. Describe the application of physics principles that can be used to determine the direction of motion and frequency of the photon that exists after the collision.

$E = hf$

We know that during the collision, momentum and energy must be conserved. Because there was no momentum in the vertical direction before the collision, the net momentum of the electron and photon must have no vertical component. Thus, the photon must have a component in the vertical direction that is downward.

By conservation of energy, the photon must have lost some energy because the electron gained kinetic energy. Thus, the frequency of the photon must be lower due to the lower energy.