Begin your response to **QUESTION 1** on this page.

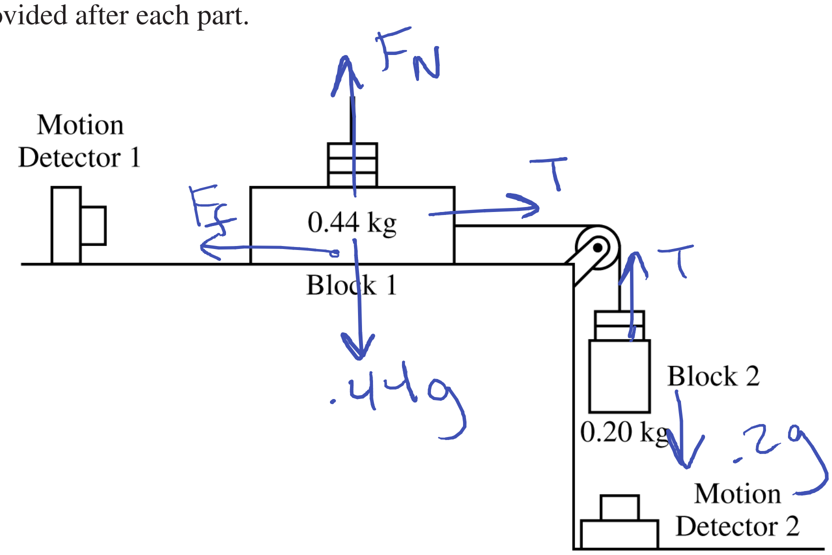
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. Students design an experiment using blocks of adjustable mass to investigate friction using the setup shown. Block 1 of initial mass 0.44 kg is placed on a rough horizontal surface and connected by a string to block 2 of initial mass 0.20 kg. The string extends over a pulley that has negligible mass and friction.

(a) Calculate the minimum value of the coefficient of static friction μ_s that would keep the two-block system at rest.

$$T = 0.2g = F_f = \mu_s F_N = \mu_s(.44g)$$

$$\mu_s = \frac{0.2}{.44} = 0.45$$

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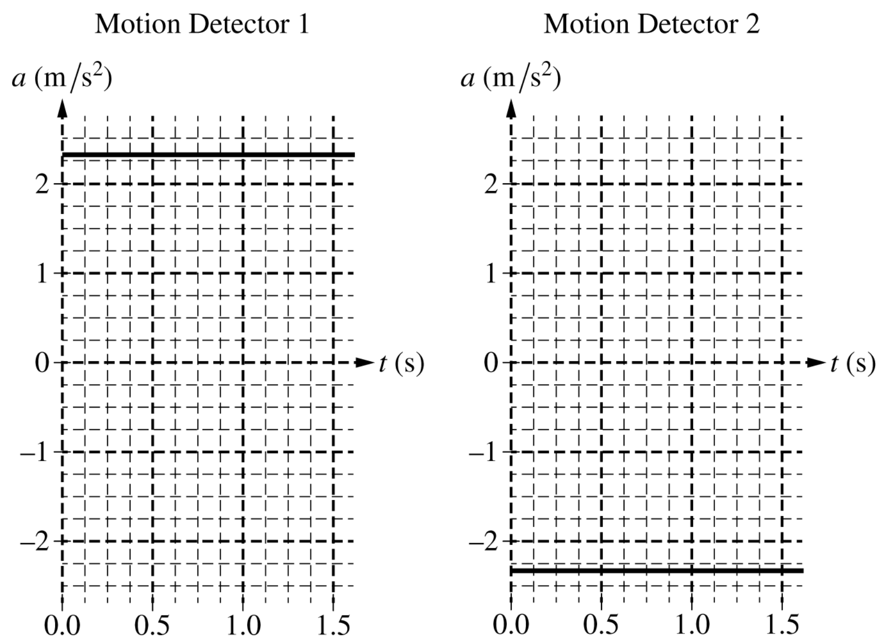
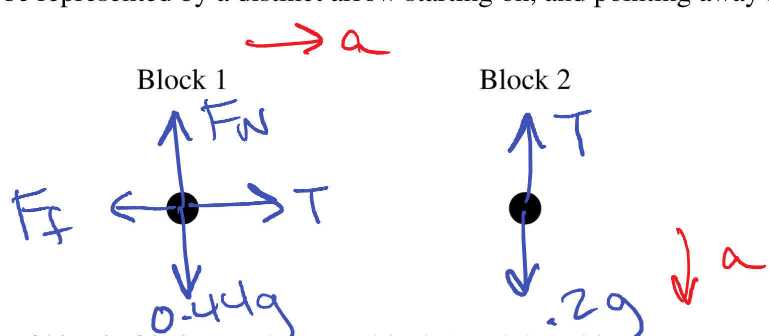
The coefficient of friction is such that when block 2 is released from rest, block 1 travels across the surface. The acceleration a of each block is recorded with motion detectors 1 and 2, as shown in the figure. The data for the motion detectors as functions of time t are shown on the graphs. For each motion detector, the positive direction is away from the detector.



(b) On the dots below, which represent the blocks, draw and label the forces (not components) that act on each block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(c) Calculate the coefficient of kinetic friction μ_k between block 1 and the table.

$a = 2.2$

$F_f = .552\text{ N}$

$= \mu_k(.44g)$

$\mu_k = 0.128$

$.2g - T = .2a \Rightarrow T = .2g - .2a$

$T - F_f = 0.44a$

$F_f = T - .44a = .2g - .2a - .44a$

$= .2g - .64a$

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(d) Careful measurements determine that the coefficient of kinetic friction is larger than the value calculated in part (c). Does the following explanation sufficiently account for the observed discrepancy?

"The horizontal table was not perfectly level before the experiment was conducted. The observed difference in the angle accounts for the difference in the expected and calculated values of μ_k."

_✓_ Yes ______ No

Justify your answer.

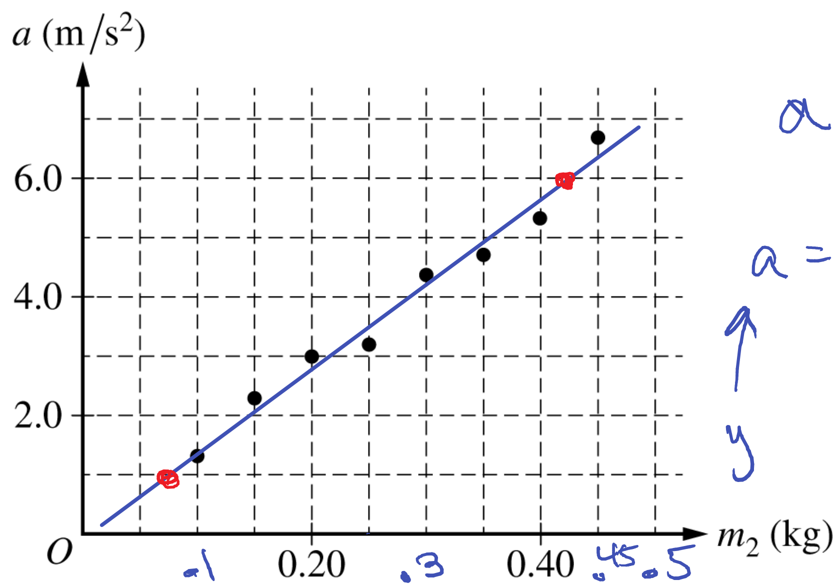
$T - F_f = ma$ $\quad T = \text{const}$

$T - \mu F_N = ma$ $\quad a = \text{increases}$

$\mu = \frac{T - ma}{F_N}$ $\quad F_N = \text{increases}$

μ changes

The experiment is moved to a surface with negligible friction and run for eight trials. In each trial, the students vary the masses m_1 and m_2 of blocks 1 and 2, respectively, while keeping the total mass $(m_1 + m_2) = 0.64$ kg constant. The data for the acceleration a of block 1 as a function of m_2 are shown on the graph below.



$a = \frac{m_2}{m_1 + m_2} g$

$a = \frac{g}{m_1 + m_2} m_2$

a → y, $\frac{g}{m_1+m_2}$ → slope, m_2 → x

(e)

i. Draw a best-fit line for the data points.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

slope $= \frac{g}{m_1 + m_2} = \frac{6 - 1}{.42 - .075} = 14.5$

$g = 9.3 \text{ m/s}^2$

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(f) The students lift the left end of the surface so that the surface is inclined at an angle to the horizontal, and the experiment for $m_2 = 0.20\,\text{kg}$ is repeated. Would the acceleration of the system be greater than, less than, or equal to the acceleration of the system in the original experiment?

__✓__ Greater than ______ Less than ______ Equal to

Justify your claim.

m_1 would now have a component of gravity in the direction of ~~a~~ acceleration.

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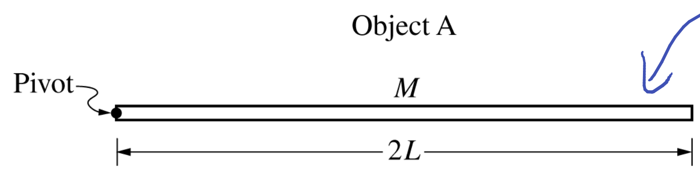
Begin your response to **QUESTION 2** on this page.

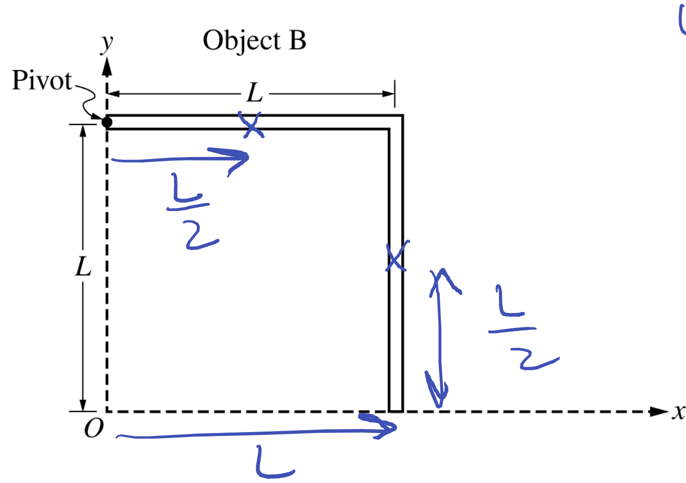


$\lambda = \frac{M}{2L}$

2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.

$$I = \int x^2\,dm = \int_0^{2L} x^2 \cdot \frac{M}{2L}\,dx = \frac{M}{2L}\frac{1}{3}x^3\Big|_0^{2L} = \frac{M}{6L}(2L)^3 = \frac{4}{3}ML^2$$



Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L.

(b) Determine the following for the given coordinate system shown in the figure.

i. The x-coordinate of the center of mass of object B

$$x_{cm} = \frac{\sum x_i m_i}{m_{tot}} = \frac{\frac{L}{2}\frac{M}{2} + L\frac{M}{2}}{M} = \frac{3}{4}L$$

ii. The y-coordinate of the center of mass of object B

$$y_{cm} = \frac{\frac{L}{2}\frac{M}{2} + L\frac{M}{2}}{M} = \frac{3}{4}L$$

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Object B has a rotational inertia of I_B about its pivot.

(c) Is the value of I_B greater than, less than, or equal to I_A ?

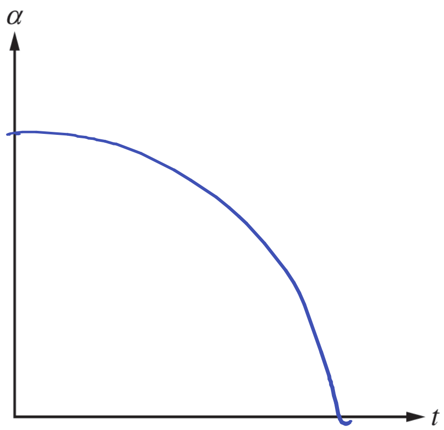
_____ Greater than   ✓ Less than   _____ Equal to

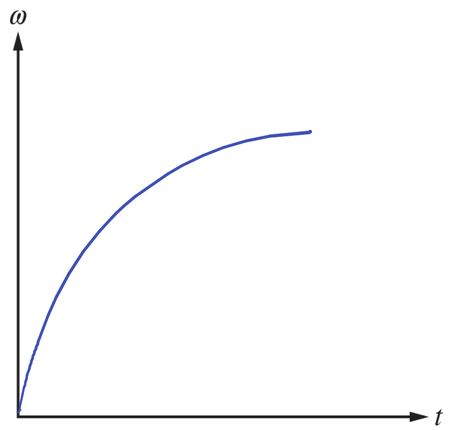
Justify your answer.

mass is distributed closer to the axis of rotation

Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.





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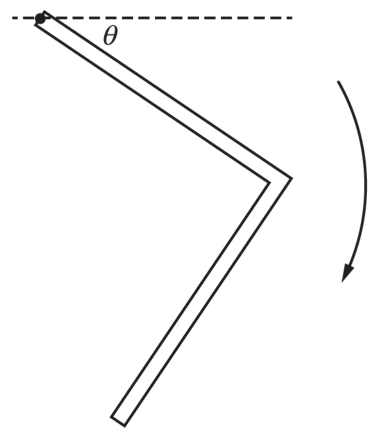
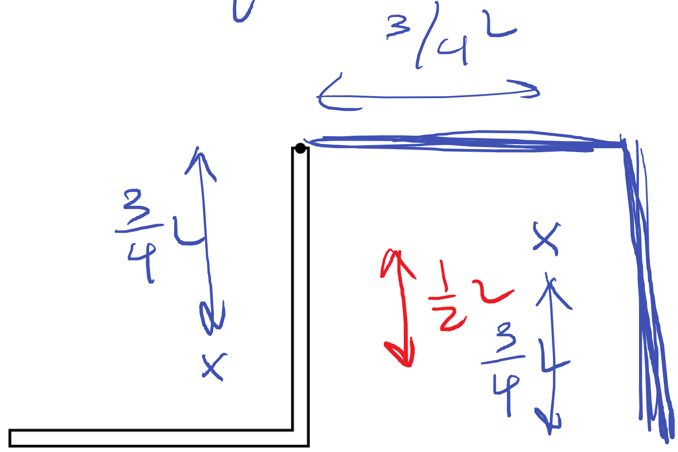


(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

_____ Increasing   ✓ Decreasing   _____ Not changing

Justify your answer. less torque as it rotates



Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

$$\frac{1}{2} I_B \omega^2 = mgh = Mg\frac{L}{2}$$

$$\omega = \sqrt{\frac{MgL}{I_B}}$$

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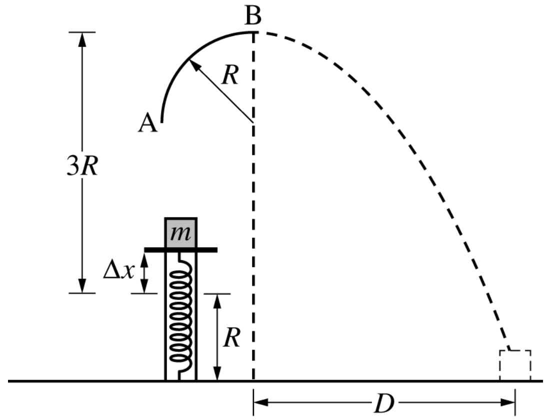
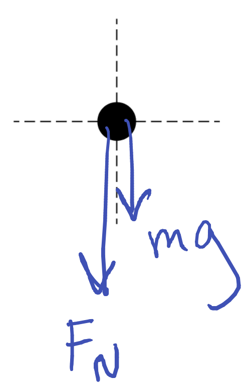


Note: Figure not drawn to scale.

3. A block of mass m is placed on top of an ideal spring of spring constant k. The block is pushed against the spring, compressing the spring a distance Δx. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



Justify your choice of vectors.

normal force perpendicular to surface

gravity is down

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$$\frac{1}{2}k\Delta x^2 = mg(3R) + \frac{1}{2}mv^2$$

(b)

i. Derive an expression for the speed v of the block at point B.

$$v = \sqrt{\frac{k}{m}\Delta x^2 - 6gR}$$

ii. Derive an expression for the magnitude of the net force F on the block at point B.

$$F_{net} = ma_c = \frac{mv^2}{R} = \frac{k\Delta x^2}{R} - 6mg$$

(c) Derive an expression for the minimum value of Δx_{min} required in order for the block to maintain contact with the track through point B.

$$F_N = 0 \Rightarrow F_{net} = mg = \frac{k}{R}\Delta x^2 - 6mg$$

The procedure is repeated several times with the distance $\Delta x > \Delta x_{min}$.

$$7mg = \frac{k}{R}\Delta x^2$$

$$\Delta x = \sqrt{\frac{7mgR}{k}}$$

(d) Calculate the distance D that the block travels.

$$\Delta y = 4R$$
$$v_{0y} = 0$$
$$a = g$$

$$\Delta y = v_{0y}t + \frac{1}{2}at^2$$
$$4R = \frac{1}{2}gt^2$$
$$t = \sqrt{\frac{8R}{g}}$$

$$\Delta x = v_{0x}t + \frac{1}{2}at^2 \quad 0$$

$$D = \sqrt{\frac{k}{m}\Delta x^2 - 6gR} \cdot \sqrt{\frac{8R}{g}}$$

$$D = \sqrt{\frac{8kR}{mg}\Delta x^2 - 48R^2}$$

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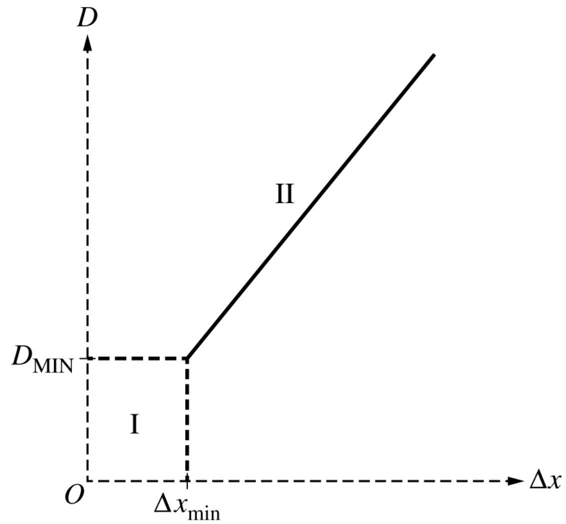
(e) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx.



i. Explain why there are no data for section I of the graph.

not enough compression for the block to go around the top curve.

ii. Explain the reason for the shape and minimum value of section II on the graph.

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