Begin your response to **QUESTION 1** on this page.

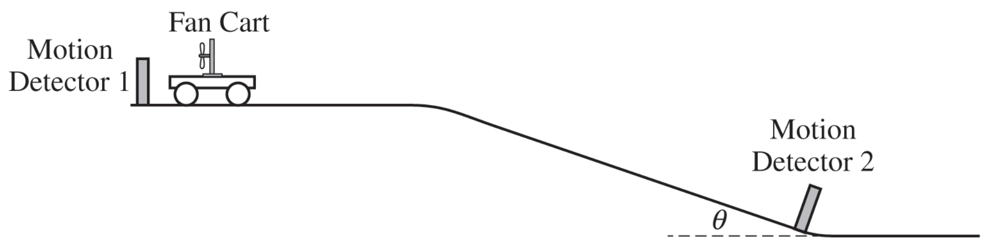
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

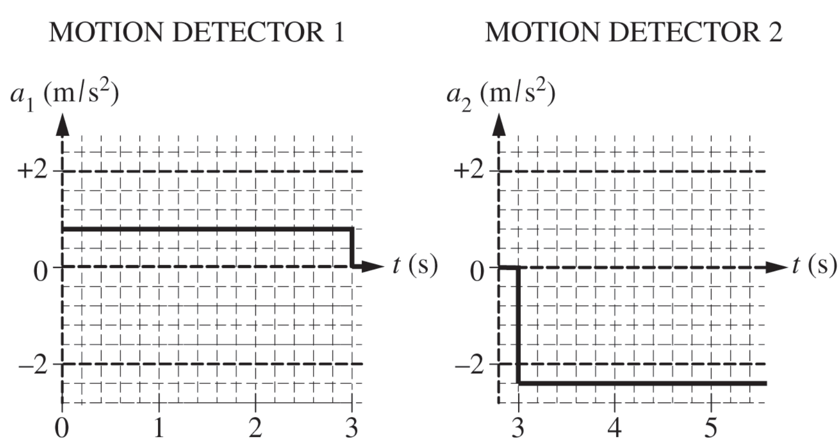
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

1. A 0.50 kg fan cart is placed on a level, horizontal track of negligible friction, as shown. The fan is turned on, and the fan cart is released from rest and moves to the right. The cart travels along the horizontal track and then down an incline. Motion detector 1 measures the acceleration a of the cart from time $t = 0$ to $t = 3$ s. At $t = 3$ s, the cart makes a smooth transition to the incline, and motion detector 2 measures the acceleration of the cart after $t = 3$ s. The fan exerts the same magnitude of force on the cart during the entire motion. The graphs below show a as functions of t. For each motion detector, the positive direction is away from the detector.



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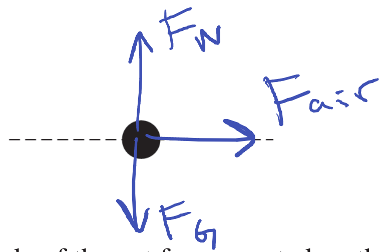
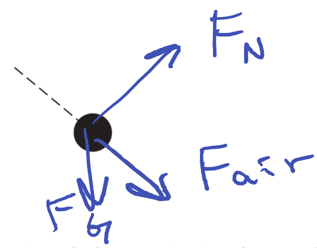
Continue your response to **QUESTION 1** on this page.

(a) On the dots below that represent the cart at two different locations, draw and label the forces (not components) that act on the cart at each location. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Cart on Horizontal Track



Cart on Incline



(b) Calculate the magnitude of the net force exerted on the fan cart when it is on the horizontal track.

x: $F_{net} = ma$

$F_{air} = ma_x = 0.5\,kg \times 0.8\,m/s^2 = 0.4\,N$

(c) Calculate the angle θ of the incline.

x $\quad F_{net} = ma$

$mg\sin\theta + F_{air} = ma$

$0.5 \times 9.8 \sin\theta + 0.4 = 0.5 \times 2.4\,m/s^2$

$\sin\theta = .163 \qquad \theta = \sin^{-1}(.163) = 9.4°$

(d) Suppose careful measurement determines the angle of the incline to be 3° larger than that calculated in part (c). Consider the following explanation.

"The scale used to measure the mass of the fan cart was not calibrated properly before the measurement, and this could account for the observed difference in the angle."

Does the explanation sufficiently account for the observed discrepancy?

_____ Yes __✓__ No

Justify your answer.

the mass would cancel out in the analysis

$F_{air} = m \times 0.8$

$$\frac{m \times 9.8 \sin\theta + m \times 0.8}{m} = \frac{m \times 2.4}{m}$$

$9.8 \sin\theta + 0.8 = 2.4$

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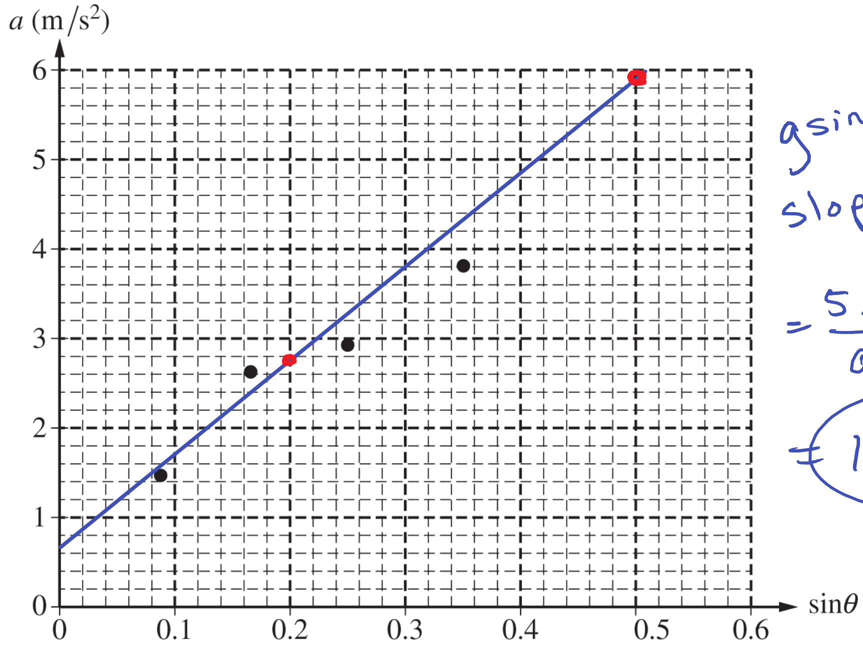
The experiment is repeated for several trials, each with a different angle for the incline. The acceleration of the cart down the incline is measured for each angle. The graph below shows the plot of the acceleration a of the cart as a function of the sine of the angle $\sin\theta$.



$g\sin\theta + a_{fan} = a$

slope $= g$

$= \frac{5.9 - 2.8}{0.5 - 0.2}$

$= 10.3\ m/s^2$

(e)

i. Draw a best-fit line for the data.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

(f) If the cart were replaced with a second cart of mass 1.0 kg that has a fan that exerts the same magnitude of force as the original fan, explain how the graph given in part (e) would change.

a_{fan} would decrease and shift the graph down vertically

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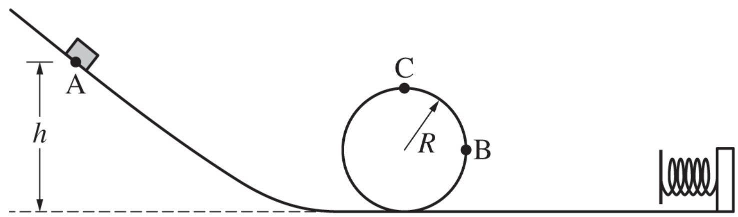
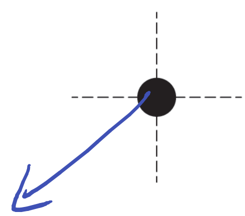
Begin your response to **QUESTION 2** on this page.



Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.



block has a component towards center of circle and downward due to gravity

Justify your answer.

(b)

i. Derive an expression for the speed v of the block at point B.

$E_0 = E_f$

$mgh = \frac{1}{2}mv^2 + mgR \Rightarrow gh - gR = \frac{1}{2}v^2$

$\boxed{v = \sqrt{2g(h-R)}}$

ii. Derive an expression for the magnitude of the net force F on the block at point B.

$F_N = ma_c = m\frac{v^2}{R} = \frac{m(2g(h-R))}{R}$

$F_G = mg$

$|F_{tot}| = \sqrt{F_N^2 + F_G^2} = \boxed{\sqrt{\left(\frac{2mg(h-R)}{R}\right)^2 + (mg)^2}}$

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(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

F_G $F_N = 0$ (barely any contact)

$mg = ma_c = m\frac{v^2}{R}$

$v^2 = Rg$

conservation of energy

$\cancel{m}gh_{min} = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(2R)$

$gh_{min} = \frac{1}{2}(Rg) + 2gR$

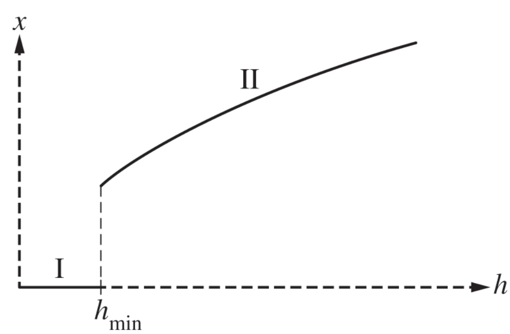
$= \frac{5}{2}Rg$

$\boxed{h_{min} = \frac{5}{2}R}$

(d) It is determined that $h = 0.30\text{ m}$ and $R = 0.10\text{ m}$. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

$mgh = \frac{1}{2}kx^2 \quad x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{\frac{mg}{2R}}} = \sqrt{4Rh} = 0.346\text{ m}$

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height h_{min} is the height calculated in part (c).



i. Explain why section I appears as a horizontal line segment on the horizontal axis.

the block doesn't make it around the loop when $h < h_{min}$

ii. Explain the reason for the shape of section II on the graph.

x has a square root dependency on h

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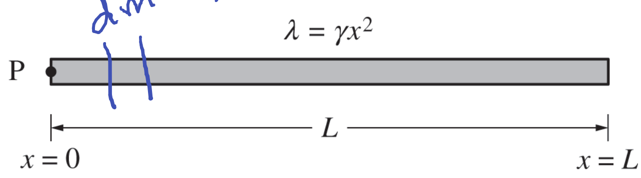
$dm = \lambda\, dx = \gamma x^2\, dx$



3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.

$$I = \int x^2\, dm = \int x^2 \lambda\, dx = \int \gamma x^4\, dx$$
$$= \frac{1}{5}x^5\gamma \Big|_{x=0}^{L} = \frac{1}{5}L^5\gamma = \frac{1}{5}L^5\left(\frac{3M}{L^3}\right)$$
$$= \boxed{\frac{3}{5}ML^2}$$

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.

$$x_{cm} = \frac{\int x\, dm}{M} = \frac{\int \gamma x^3\, dx}{M} = \frac{\frac{1}{4}x^4\gamma \Big|_{x=0}^{L}}{M} = \frac{\frac{1}{4}L^4\left(\frac{3M}{L^3}\right)}{M}$$
$$= \frac{3}{4}L$$

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

__✓__ Greater than ______ Less than ______ Equal to

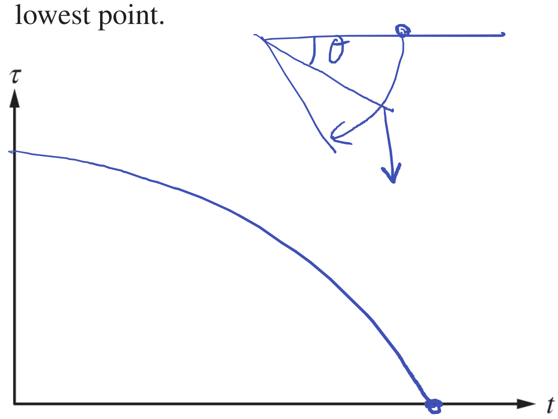
Justify your answer.

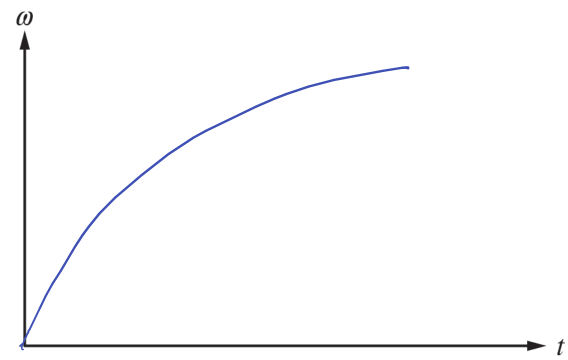
the mass is distributed further from point P than the center of mass.

Continue your response to **QUESTION 3** on this page.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.





(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing   ✓ Decreasing   _____ Not changing

Justify your answer.

$$\tau = R\cos\theta \cdot mg$$

$$\frac{d\tau}{dt} = -mgR\sin\theta < 0$$

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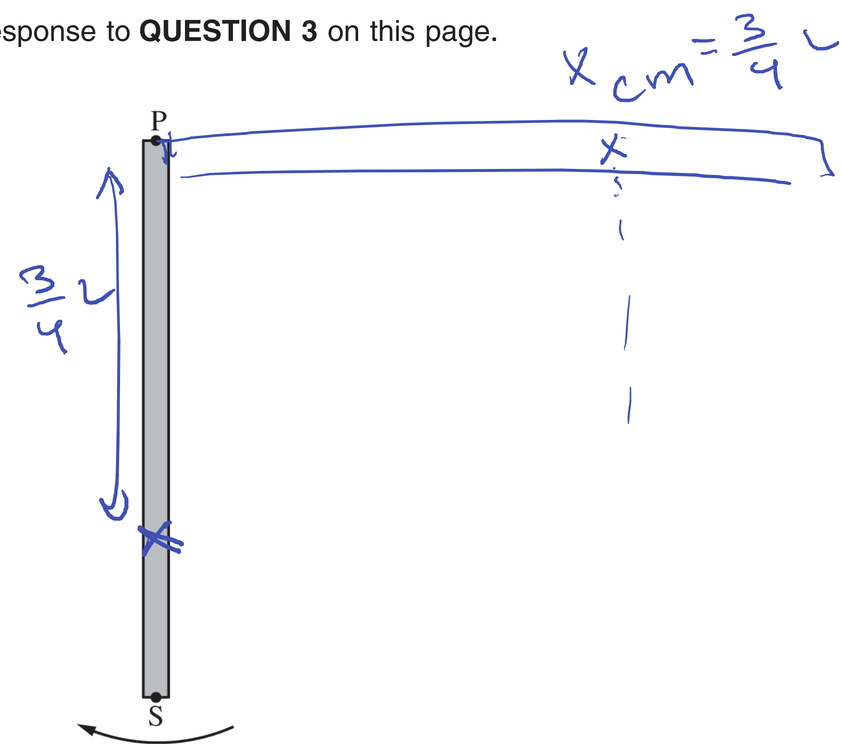

(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

$$\frac{1}{2} I \omega^2 = mgh$$

$$\frac{1}{2}\left(\frac{3}{5}\cancel{m}L^2\right)\omega^2 = \cancel{M}g\left(\frac{3}{4}L\right)$$

$$w^2 = \frac{g\left(\frac{\cancel{3}}{4}\cancel{L}\right)}{L^{\cancel{2}}} \times 2 \times \frac{5}{\cancel{3}}$$

$$w = \sqrt{\frac{5}{2}\frac{g}{L}}$$

$$v = r\omega = L\sqrt{\frac{5}{2}\frac{g}{L}} = 4.95\,\frac{m}{s}$$

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