Begin your response to **QUESTION 1** on this page.

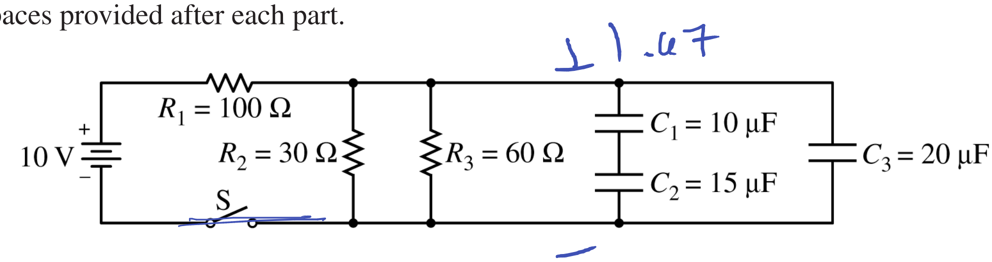
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit shown above is composed of an ideal 10 V battery, three resistors and three capacitors with the values shown, and an open switch S. The capacitors are initially uncharged. Switch S is now closed.

(a) Calculate the current through R_1 immediately after switch S is closed.

$\frac{10V}{100\Omega} = 0.1A$

Switch S has been closed for a long time, and the circuit has reached a steady state.

(b) Calculate the potential difference across R_1.

capacitors open

$I = \frac{10}{120} = 0.0833A$ $\quad V_{R_1} = IR_1 = 8.33\ V$

(c)

i. Calculate the charge stored on the positive plate of capacitor C_2.

$Q = C_{eq}V$

$= 6\mu F \times 1.67V$

$= 10\mu C$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{150}{25} = 6\mu F$

ii. Is the charge stored on capacitor C_3 greater than, less than, or equal to the charge stored on capacitor C_2 ?

✓ Greater than _____ Less than _____ Equal to

Justify your answer.

$Q_3 = 1.67V \times 20\mu F = 33.3\mu C$

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Switch S is then opened.

(d)

i. Determine the current through R_1 immediately after the switch is opened.

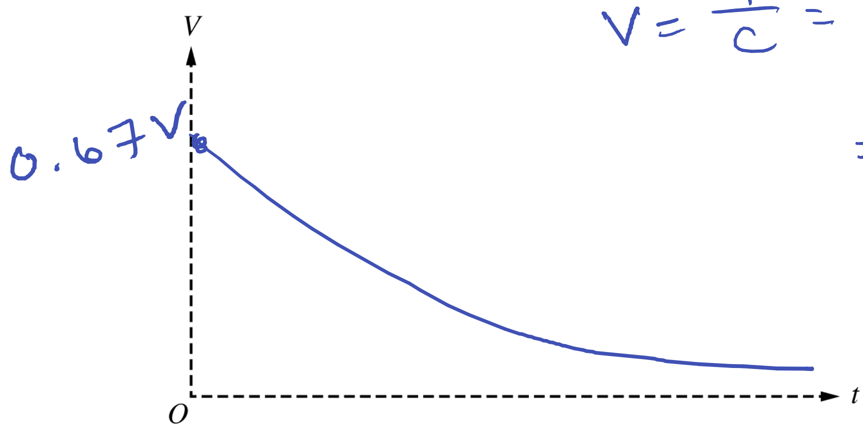
0 A

ii. Calculate the current through R_2 immediately after the switch is opened.

$I = \frac{V}{R} = \frac{1.67\,V}{30\,\Omega} = 0.0556\,A$

(e) On the axes below, sketch a graph of the potential difference V across capacitor C_2 as a function of time t if switch S is opened at time $t = 0$. Label the maximum value.

$V = \frac{Q}{C} = \frac{10\,\mu C}{15\,\mu F} = 0.67\,V$



Capacitor C_3 is replaced by two $10\,\mu F$ capacitors connected in series, switch S is closed, and the circuit reaches equilibrium. Switch S is then opened at time $t = 0$.

(f) For $t > 0$, would the sketch of a graph of the new voltage across C_2 as a function of time be above, below, or the same as the sketch for part (e) ?

_____ Above ✓ Below _____ The same

Justify your answer.

Same voltage V but RC time constant is smaller → faster drop to 0V

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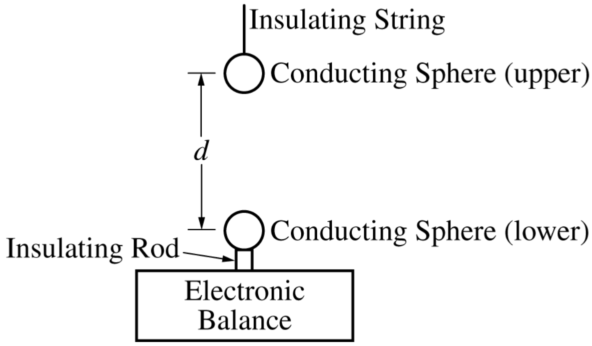
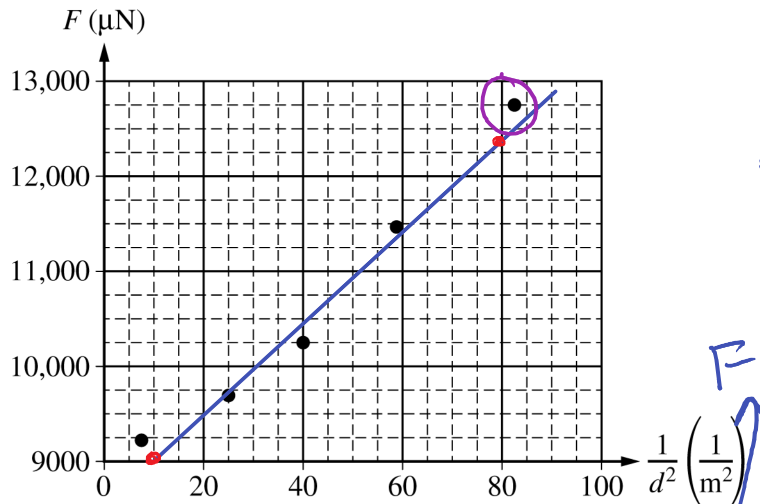


2. Students perform an experiment to study the force between two charged objects using the apparatus shown above, which contains two identical conducting spheres. The upper sphere is attached to an insulating string, which can be used to move the sphere downward. The lower sphere sits on an insulating rod, which is on an electronic balance. The electronic balance is zeroed before the lower sphere and insulating rod are in place.

For the first trial, a charge of Q is placed on each sphere and then the upper sphere is slowly moved downward. The students measure the distance d between the centers of the spheres and the magnitude F of the force that appears on the electronic balance. The recorded data are shown on the graph of F as a function of $\frac{1}{d^2}$ shown below.



$F = \frac{k q_1 q_2}{r^2}$

$F = kQ^2 \cdot \frac{1}{r^2}$

y slope x

(a)

i. Draw a line that represents the best fit to the points shown.

ii. Use the graph to calculate the charge Q.

$\text{slope} = \frac{12375\,\mu N - 9000\,\mu N}{80 - 10} = 9\times10^9 Q^2$

$Q = 73.1\,nC$

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iii. On the graph on the previous page, draw a circle around the data point that was taken when the distance between the centers of the spheres was the least.

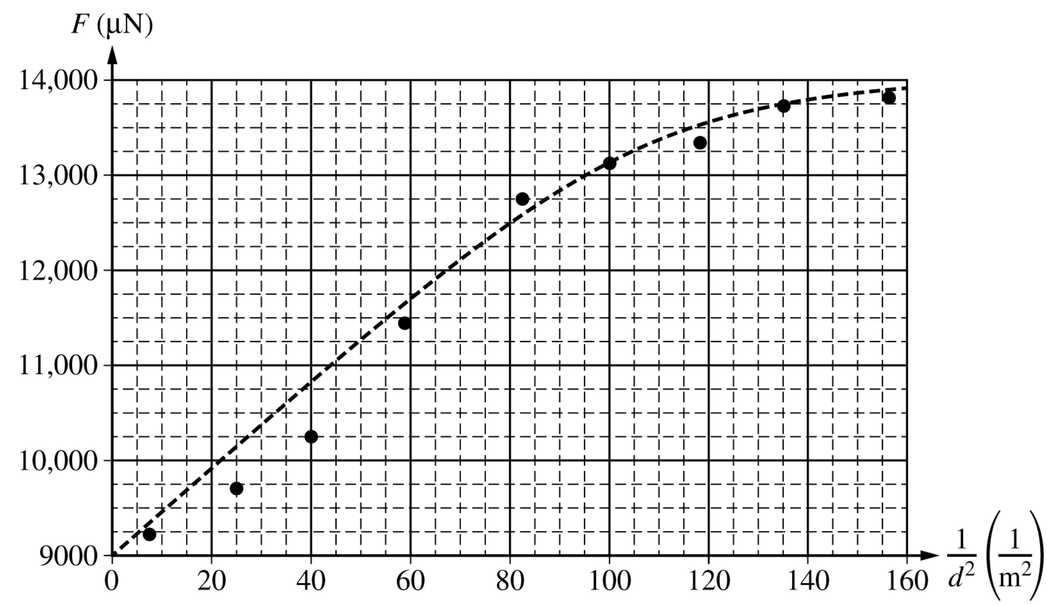
iv. Determine the distance between the centers of the spheres for the data point indicated above.

$\frac{1}{d^2} = 82.5$ $\boxed{d = .11\text{m}}$

v. What physical quantity does the vertical intercept represent?

weight of insulating rod + sphere

Justify your answer.



The experiment is extended by collecting additional data points, which appear on the right side of the graph shown above. The new data points do not follow the linear pattern seen with the first points. The group of students tries to explain this discrepancy.

(b) One student suspects that charge is slowly leaking off the top sphere. Could this explain the discrepancy?

__✓__ Yes ______ No

Justify your answer.

Q is less at closer distances ⇒ lower F when $\frac{1}{d^2}$ is larger

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(c) A second student suspects that the excess charges have rearranged themselves, polarizing the spheres.

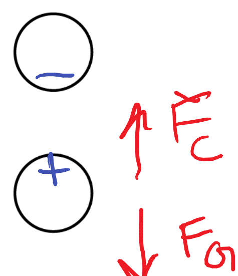
i. On the circles representing the spheres below, use a single "+" sign on each sphere to represent the locations of highest concentration of the excess positive charges.

ii. Explain how this rearrangement could be responsible for the discrepancy.

F reduces at low distances because charges redistribute themselves further away

(d) A third student suggests that the experiment be modified so that the top sphere is given a negative charge that is equal in magnitude to the positive charge given to the bottom sphere.

i. On the circles representing the spheres below, use a single "+" sign on the bottom sphere to represent the location of highest concentration of the excess positive charges. Use a single "–" sign on the top sphere to represent the location of the highest concentration of the excess negative charges.



ii. For a separation distance equal to that of the data point indicated in part (a)(iii), would the magnitude of the force reading with spheres of opposite charges be greater than, less than, or equal to the magnitude of the force reading with spheres of the same charges?

_____ Greater than   ✓ Less than   _____ Equal to

Justify your answer.

the F_c would be pointing up due to attraction. This would make the net force smaller since net force is down

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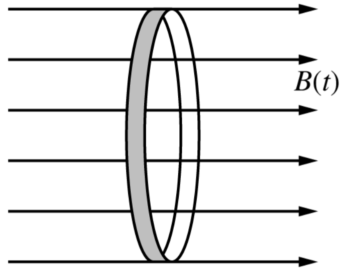
Area of Ring = A
Resistance of Ring = R



3. A thin, conducting ring of area A and resistance R is aligned in a uniform magnetic field directed to the right and perpendicular to the plane of the ring, as shown. At time $t = 0$, the magnitude of the magnetic field is B_0. At $t = 1$ s, the magnitude of the magnetic field begins to decrease according to the equation $B(t) = \frac{\beta}{t}$, where β has units of T·s.

(a) Derive an equation for the magnitude of the induced current I in the ring as a function of t for $t > 1$ s. Express your answer in terms of β, A, R, t, and physical constants, as appropriate.

$$\Phi = BA = \frac{\beta}{t} \cdot A \qquad I = \frac{V}{R} = -\frac{\beta A}{Rt^2}$$

$$V = \frac{d\Phi}{dt} = -\frac{\beta A}{t^2}$$

$$\frac{\beta A}{Rt^2}$$

Assume $A = 0.50\,\text{m}^2$, $R = 2.0\,\Omega$, and $\beta = 0.50\,\text{T}\cdot\text{s}$.

(b) Calculate the electrical energy dissipated in the ring from $t = 1$ s to $t = 2$ s.

$$P = I^2R = \frac{\beta^2A^2}{R^2t^4}R = \frac{\beta^2A^2}{R}\frac{1}{t^4}$$

$$E = \int_1^2 P\,dt = \int_1^2 \frac{\beta^2A}{R}t^{-4}\,dt$$

$$= \frac{\beta^2A}{R}\frac{t^{-3}}{-3}\Big|_1^2 = \frac{\beta^2A}{3R}\left(1-\frac{1}{8}\right)$$

$$= .00911\,\text{J}$$

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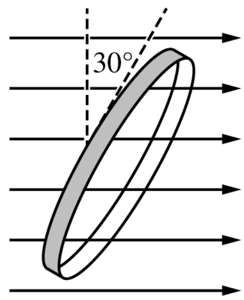


The ring is then rotated so that the plane of the ring is aligned at a 30° angle to the magnetic field, as shown.

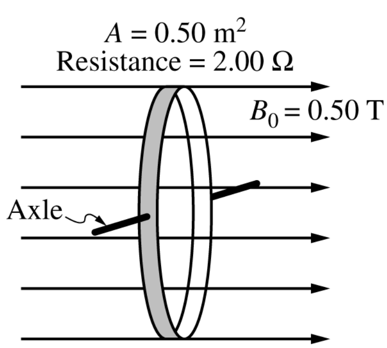
The magnitude of the magnetic field is reset to a magnitude of B_0 at a new time $t = 0$ and again begins to decrease at $t = 1$ s according to the equation $B(t) = \dfrac{\beta}{t}$, where β has units of T·s.

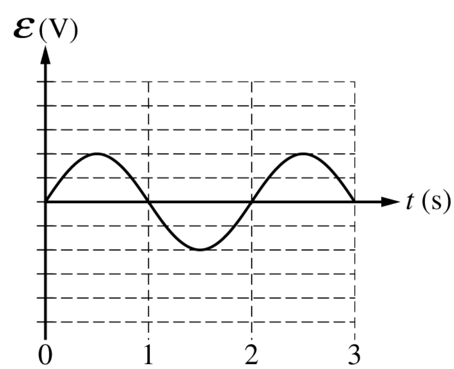
(c) Will the amount of energy dissipated in the ring from $t = 1$ s to $t = 2$ s be greater than, less than, or equal to the energy dissipated in part (b) ?

_____ Greater than ___✓___ Less than _____ Equal to

Justify your answer.

less Area due to B field not being perpendicular to surface





The ring is now mounted on an axle that is perpendicular to the magnetic field. The magnitude of the magnetic field is now held at a constant $B_0 = 0.50$ T, as shown. The ring rotates about the axle, and the emf ε induced in the ring as a function of time t is shown on the graph.

(d) Calculate the angular speed ω of the rotating ring in rad/s.

$T = 2s$

$f = \frac{\omega}{2\pi} = \frac{1}{T}$

$\omega = \frac{2\pi}{T} = \pi \frac{rad}{s}$

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Continue your response to **QUESTION 3** on this page.

(e) Calculate the magnitude of the maximum emf ε_{MAX} induced in the ring.

$B_0 A \omega$
$=$

$V = \frac{d\Phi}{dt}$

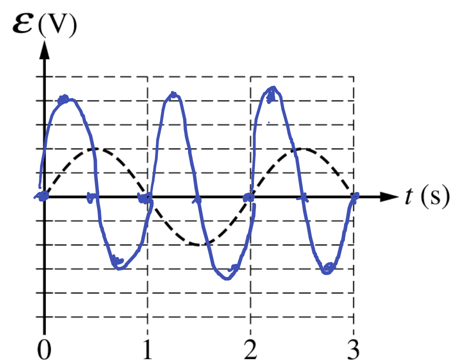
$\Phi = BA = B_0 A \cos\theta$

$\left|\frac{d\Phi}{dt}\right| = B_0 A \sin\theta \cdot \frac{d\theta}{dt}$

$= B_0 A \omega \sin\theta \Rightarrow$

The ring now begins to rotate at an angular speed 2ω.

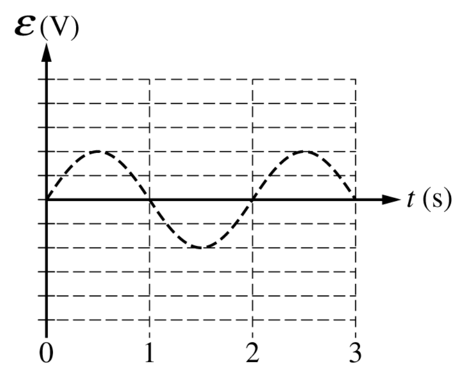
(f) On the graph below, draw a curve to indicate the new induced emf ε in the ring. The dashed curve shows the emf induced under the original conditions.



Justify your sketch, specifically identifying and addressing any similarities or differences between the sketch and the original graph.

still sinusoid w/ 2x amplitude and ½ frequency

PRACTICE GRAPH - Use the graph below to practice your sketch for part (f). Any work shown on the graph below will NOT be graded.



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