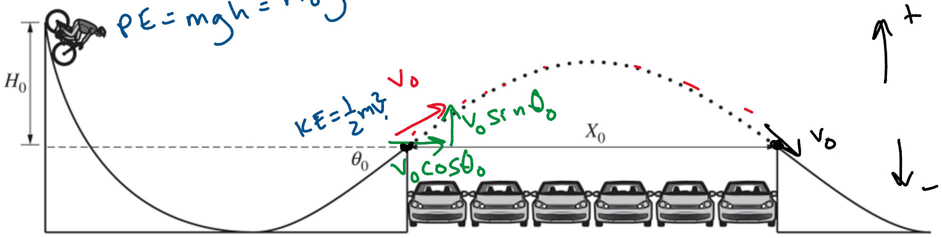
2021

Friday, May 07, 2021 - 12:05 PM



Note: Figure not drawn to scale.

$\cancel{m_0} g H_0 = \frac{1}{2} \cancel{m_0} v_0^2$

$v_0^2 = 2 g H_0$

1. (7 points, suggested time 13 minutes)

A stunt cyclist builds a ramp that will allow the cyclist to coast down the ramp and jump over several parked cars, as shown above. To test the ramp, the cyclist starts from rest at the top of the ramp, then leaves the ramp, jumps over six cars, and lands on a second ramp.

H_0 is the vertical distance between the top of the first ramp and the launch point.
θ_0 is the angle of the ramp at the launch point from the horizontal.
X_0 is the horizontal distance traveled while the cyclist and bicycle are in the air.
m_0 is the combined mass of the stunt cyclist and bicycle.

(a) Derive an expression for the distance X_0 in terms of H_0, θ_0, m_0, and physical constants, as appropriate.

x	y
$\Delta x = X_0$	$\Delta x = 0$
$v_0 = v_0 \cos\theta_0$	$v_0 = v_0 \sin\theta_0$
$v = v_0 \cos\theta_0$	$v = ?$
$a = 0$	$a = -g$
$t = ?$	$t =$

$\Delta x = v_0 t + \frac{1}{2} a t^2$

$0 = v_0 \sin\theta_0 t - \frac{1}{2} g t^2$

$0 = t\left(v_0 \sin\theta_0 - \frac{1}{2} g t\right)$

~~$t = 0$~~ $\quad v_0 \sin\theta_0 = \frac{1}{2} g t$

$t = \frac{2 v_0 \sin\theta_0}{g}$

$\Delta x = v_0 t + \cancel{\frac{1}{2} a t^2} \rightarrow 0$

$X_0 = v_0 \cos\theta_0 t$

$X_0 = v_0 \cos\theta_0 \frac{2 v_0 \sin\theta_0}{g}$

$= \frac{2 \sin\theta_0 \cos\theta_0}{g} v_0^2$

$\frac{2 \sin\theta_0 \cos\theta_0}{\cancel{g}} 2 \cancel{g} H_0 = \boxed{4 H_0 \sin\theta_0 \cos\theta_0}$

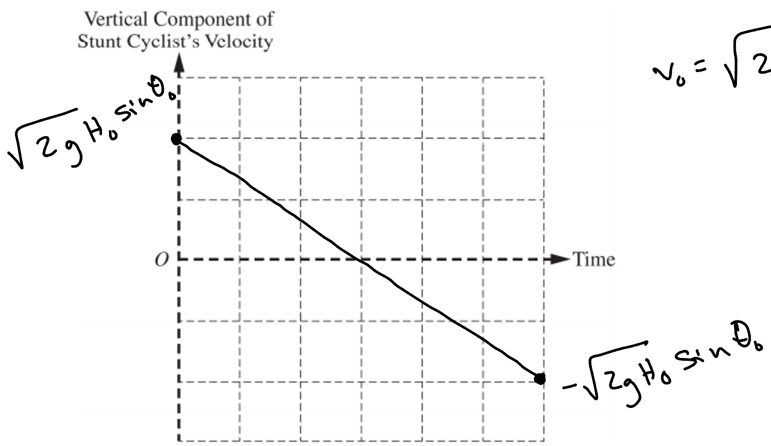
(b) If the vertical distance between the top of the first ramp and the launch point were $2H_0$ instead of H_0, with no other changes to the first ramp, what is the maximum number of cars that the stunt cyclist could jump over? Justify your answer, using the expression you derived in part (a).

doubling H_0 to $2H_0$

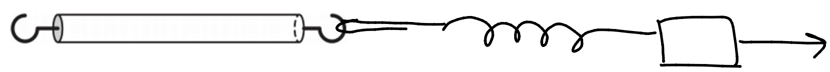
X_0 = double

double # cars = 12

(c) On the axes below, sketch a graph of the vertical component of the stunt cyclist's velocity as a function of time from immediately after the cyclist leaves the ramp to immediately before the cyclist lands on the second ramp. On the vertical axis, clearly indicate the initial and final vertical velocity components in terms of H_0, θ_0, m_0, and physical constants, as appropriate. Take the positive direction to be upward.



$v_0 = \sqrt{2gH_0} \sin\theta_0$



2. (12 points, suggested time 25 minutes)

A group of students is investigating how the thickness of a plastic rod affects the maximum force F_{max} with which the rod can be pulled without breaking. Two students are discussing models to represent how F_{max} depends on rod thickness.
Student A claims that F_{max} is directly proportional to the radius of the rod.
Student B claims that F_{max} is directly proportional to the cross-sectional area of the rod—the area of the base of the cylinder, shaded gray in the figure above.

(a) The students have a collection of many rods of the same material. The rods are all the same length but come in a range of six different thicknesses. Design an experimental procedure to determine which student's model, if either, correctly represents how F_{max} depends on rod thickness.

In the table below, list the quantities that would be measured in your experiment. Define a symbol to represent each quantity, and also list the equipment that would be used to measure each quantity. You do not need to fill in every row. If you need additional rows, you may add them to the space just below the table.

Quantity to be Measured	Symbol for Quantity	Equipment for Measurement
radius of rod	r	ruler
force applied	F	spring scale

Describe the overall procedure to be used, referring to the table. Provide enough detail so that another student could replicate the experiment, including any steps necessary to reduce experimental uncertainty. As needed, use the symbols defined in the table and/or include a simple diagram of the setup.

1) measure the radius of all the rods

2) attach one end of rod to the wall or anchor (fixed)

scale

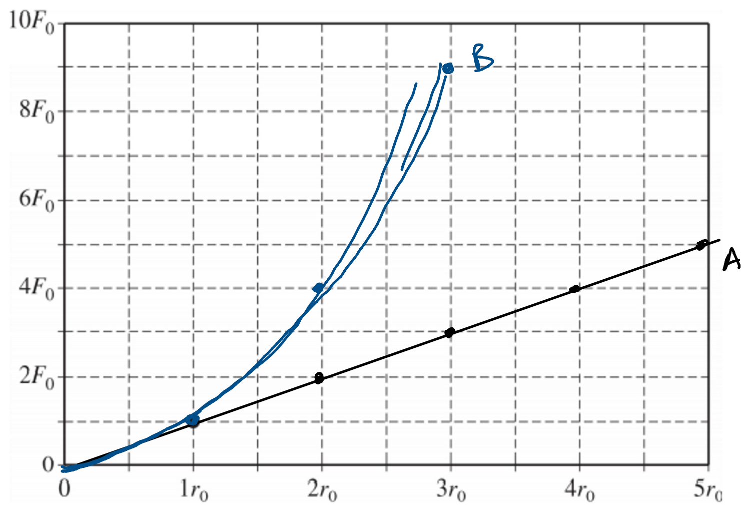
2) attach one end of rod to the wall or anchor (fixed)

3) attach other end of rod to a spring scale

4) pull on scale slowly and record the force until it breaks (video recording with a clear shot of the spring scale measurment)

5) repeat with all rods

(b) For a rod of radius r_0, it is determined that F_{max} is F_0, as indicated by the dot on the grid below. On the grid, draw and label graphs corresponding to the two students' models of the dependence of F_{max} on rod radius. Clearly label each graph "A" or "B," corresponding to the appropriate model.



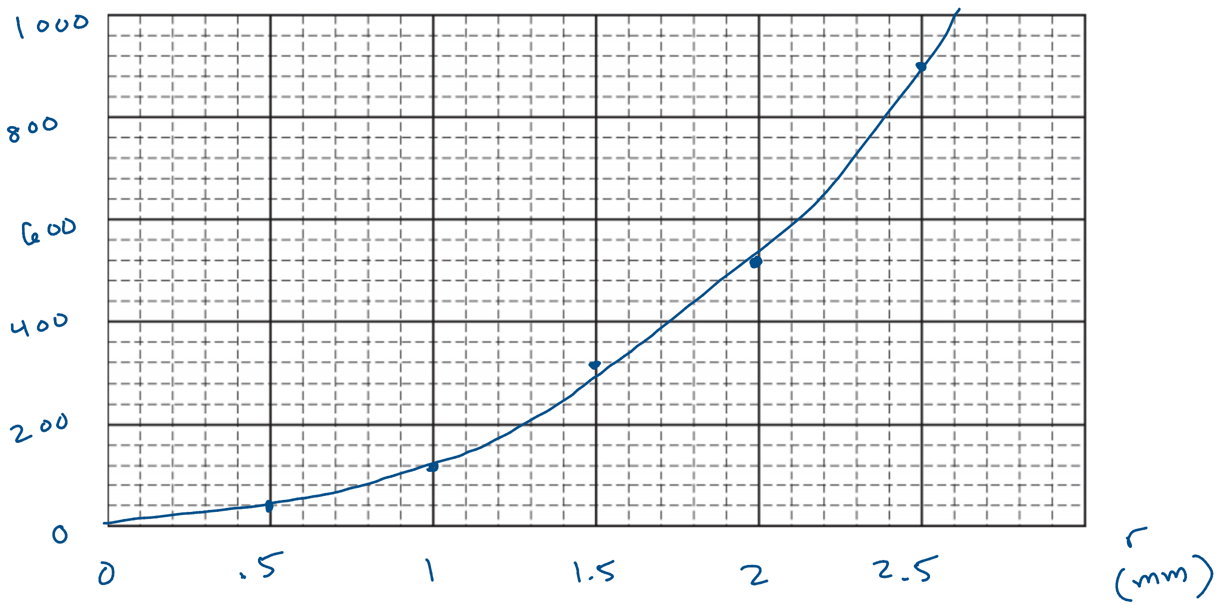
A: $F_{max} = k_A r$

B: $F_{max} = k_B \pi r^2$

The table below shows results of measurements taken by another group of students for rods of different thicknesses.

Rod radius (mm)	0.5	1.0	1.5	2.0	2.5
F_{max} (N)	40	120	320	520	900

(c) On the grid below, plot the data points from the table. Clearly scale and label all axes, including units. Draw either a straight line or a curve that best represents the data.



(d) Which student's model is more closely represented by the evidence shown in the graph you drew in part (c) ?

____ Student A's model: F_{max} is directly proportional to the radius of the rod.

✓ Student B's model: F_{max} is directly proportional to the cross-sectional area of the rod.

Explain your reasoning.

the ~~shape~~ relationship ~~of the~~ between F and r looks more quadratic than linear

3. (12 points, suggested time 25 minutes)

(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

$$F\Delta t = F_H t_f = \Delta p = M_D \Delta v = M_D v_D$$

$$\boxed{v_D = \frac{F_H t_f}{M_D}}$$

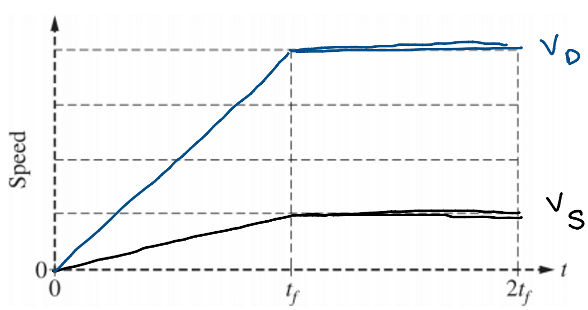
ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D / v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

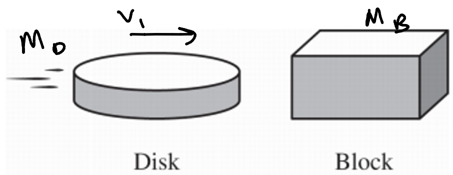
$$F_H t_f = M_S v_S$$

$$v_S = \frac{F_H t_f}{M_S}$$

$$\frac{v_D}{v_S} = \frac{\frac{\cancel{F_H t_f}}{M_D}}{\frac{\cancel{F_H t_f}}{M_S}} = \boxed{\frac{M_S}{M_D}}$$

(b) Assume that the student's mass is greater than that of the disk ($M_S > M_D$). On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.





(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

$F\Delta t = m\Delta V$

i. Suppose the mass of the disk is much greater than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

$V_{cm} \approx v_1$

very large disk, the impulse on the disk from the block is negligible compared to mass $\Rightarrow \Delta V$ is small

ii. Suppose the mass of the disk is much less than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

$V_{cm} \approx 0$

impulse on the large block is very tiny compared to its mass

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

$$M_D v_1 = (M_B + M_D) V_{cm}$$

$$\boxed{V_{cm} = \frac{M_D V_1}{M_B + M_D}}$$

iv. Consider the scenario from part (c)(i), where the mass of the disk was much greater than the mass of the block. Does your equation for v_{cm} from part (c)(iii) agree with your reasoning from part (c)(i) ?

_✓_ Yes ____ No

Explain your reasoning by addressing why, according to your equation, v_{cm} becomes (or approaches) a certain value when M_D is much greater than M_B.

$M_B + M_D \approx M_D$ for ~~very~~ $M_D >> M_B$

$$V_{cm} \approx \frac{M_D V_1}{M_D} = V_1$$

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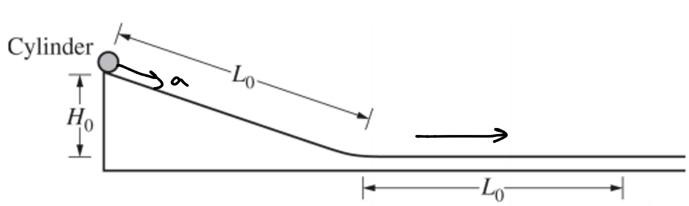
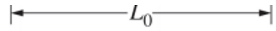


$\cancel{\frac{1}{2} m v_0}$

4. (7 points, suggested time 13 minutes)

$\cancel{v_0^2 =}$ $v^2 = v_0^2 + 2a\Delta x$



$\cancel{\frac{1}{2}mv_0}$

4. (7 points, suggested time 13 minutes) $\cancel{v_0^2} =$ $v^2 = v_0^2 + 2a\Delta x$

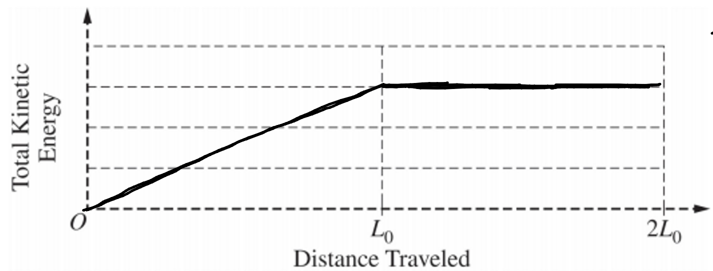
A cylinder of mass m_0 is placed at the top of an incline of length L_0 and height H_0, as shown above, and released from rest. The cylinder rolls without slipping down the incline and then continues rolling along a horizontal surface.

(a) On the grid below, sketch a graph that represents the total kinetic energy of the cylinder as a function of the distance traveled by the cylinder as it rolls down the incline and continues to roll across the horizontal surface.

0

$\frac{1}{2}mv^2 = \frac{1}{2}\cancel{mv_0^2} + ma\Delta x$

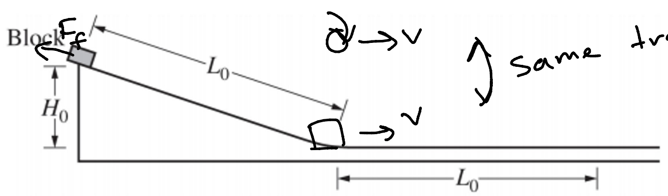
$\frac{1}{2}mv^2 = ma\Delta x$



$m_0 gH_0$ 0 rot KE

same trans KE

$\frac{1}{2}mv^2$

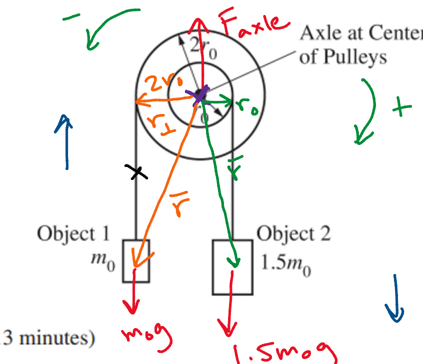


The cylinder is again placed at the top of the incline. A block, also of mass m_0, is placed at the top of a separate rough incline of length L_0 and height H_0, as shown above. When the cylinder and block are released at the same instant, the cylinder begins to roll without slipping while the block begins to accelerate uniformly. The cylinder and the block reach the bottoms of their respective inclines with the same translational speed.

(b) In terms of energy, explain why the two objects reach the bottom of their respective inclines with the same translational speed. Provide your answer in a clear, coherent paragraph-length response that may also contain figures and/or equations.

Both block and cylinder start with the same gravitational potential energy. The block loses some energy due to work done by friction, whereas the cylinder does not lose any energy. At the bottom, they both have the same amount of translational kinetic energy, but the cylinder has rotational kinetic energy. The rotational kinetic energy of the cylinder is equal to the work done by friction on the block.

Begin your response to **QUESTION 5** on this page.



$\tau = r_{\perp}F$

5. (7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

m_0 is the mass of object 1.
$1.5m_0$ is the mass of object 2.
r_0 is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0, r_0, and physical constants, as appropriate.

$$\tau_{net} = r_0(1.5m_0g) - 2r_0m_0g$$

$$= -0.5r_0m_0g$$

$$\boxed{|\tau_{net}| = 0.5r_0m_0g}$$

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

net torque is in the counterclockwise direction so

$\tau = I\alpha$ ... counterclockwise.

direction so

$\tau_{net} = I\alpha$

$\Rightarrow$ angular acceleration is counterclockwise.

(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

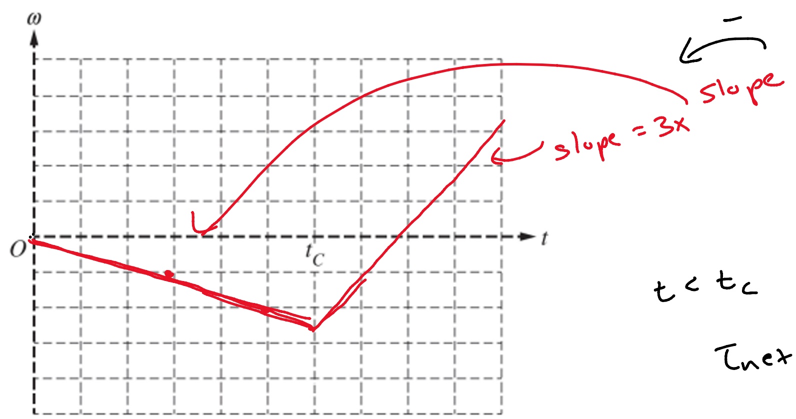
___ Same direction ✓ Opposite directions

Briefly explain your reasoning.

angular velocity is counterclockwise b/c object 1 has been falling.

when the string is cut, the net torque is clockwise $\Rightarrow$ α is clockwise

(c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.



$-$ $+$

$t < t_c$

$\tau_{net} = -0.5\, m_0 r_0 g$

3x

$t > t_c$

$\tau_{net} = +1.5\, m_0 r_0 g$